ECML PKDD 2023

KAIST

# Hypercore Decomposition for Non-Fragile Hyperedges: Concepts, Algorithms, Observations, and Applications




Fanchen Bu    Geon Lee    Kijung Shin

# Group Interactions are Everywhere

- **Ex 1: Collaboration** among researchers, e.g., co-authorship

Microsoft Academic Graph

**Hypercore decomposition for non-fragile hyperedges: concepts, algorithms, observations, and applications**

Fanchen Bu[1] · Geon Lee[2] · Kijung Shin[1,2]

**Improving the core resilience of real-world hypergraphs**

Manh Tuan Do[1] · Kijung Shin[1,2]

**Reciprocity in directed hypergraphs: measures, findings, and generators**

Sunwoo Kim[1] · Minyoung Choe[1] · Jaemin Yoo[3] · Kijung Shin[1,2]

**Datasets, tasks, and training methods for large-scale hypergraph learning**

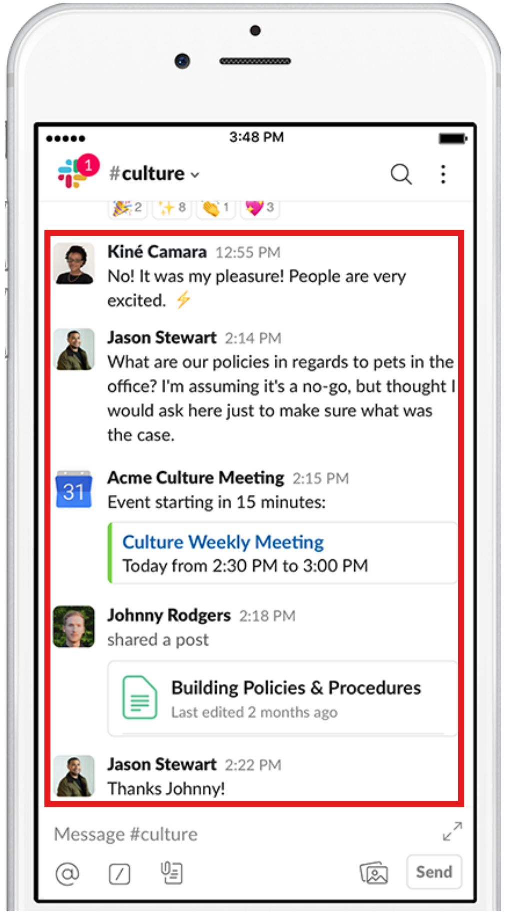
Sunwoo Kim[1] · Dongjin Lee[2] · Yul Kim[3] · Jungho Park[3] · Taeho Hwang[3] · Kijung Shin[1,2]

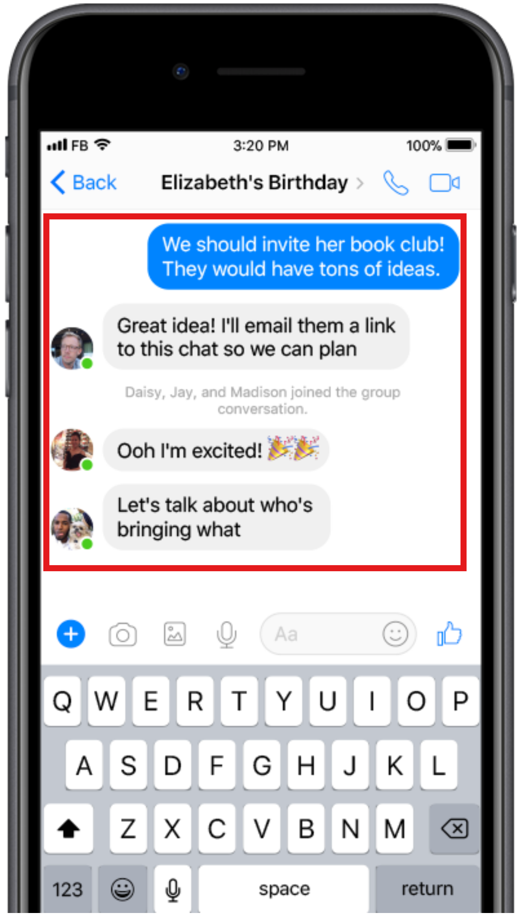
**Interplay between topology and edge weights in real-world graphs: concepts, patterns, and an algorithm**

Fanchen Bu[1] · Shinhwan Kang[2] · Kijung Shin[1,2]

# Group Interactions are Everywhere (cont.)

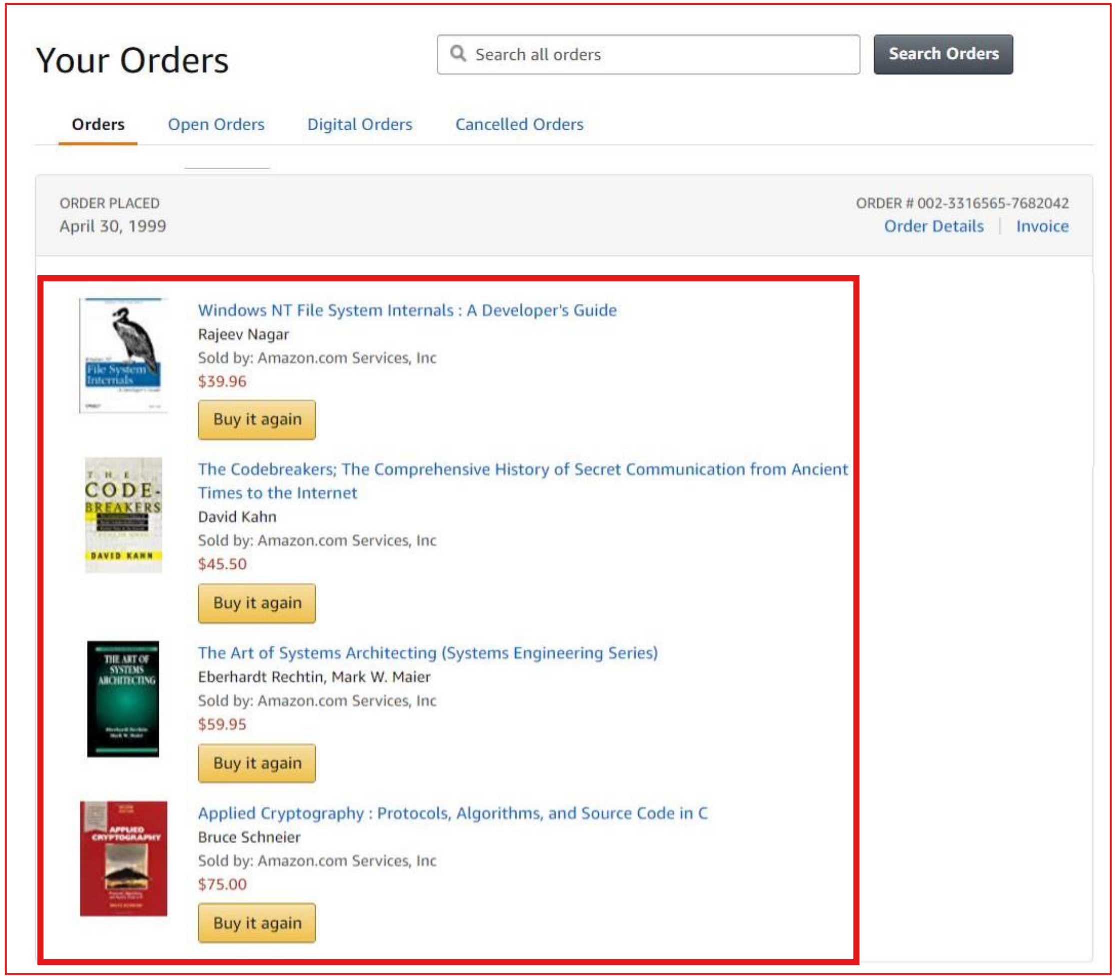
- **Ex 2: Communication** among a group, e.g., emails, online group chats




# Group Interactions are Everywhere (cont.)

- **Ex 3: Co-purchases** of items on Ecommerce platforms



# Hypergraphs: A Good Model for Group Interactions

- **Q:** How can we represent real-world group interactions?
- **Hypergraphs** model group interactions among individuals or objects
- Each **hyperedge** is a subset of any number of nodes
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| Publications (Hyperedges) |
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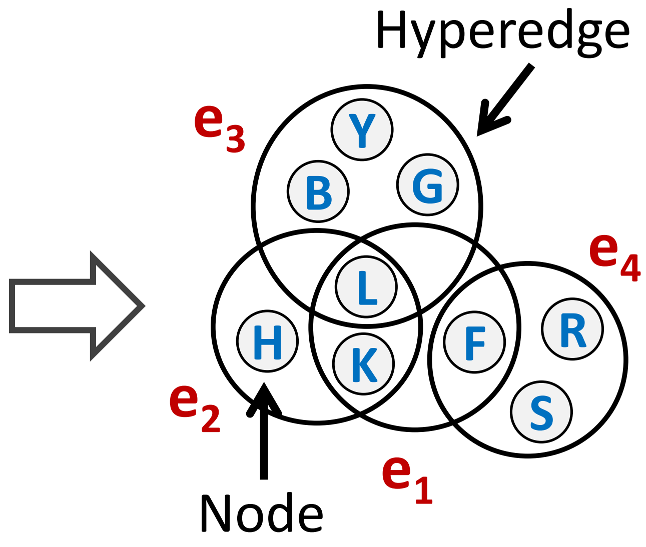

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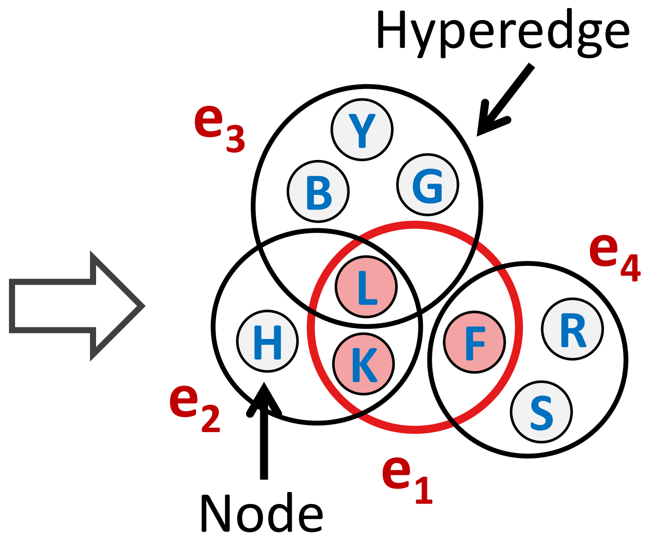

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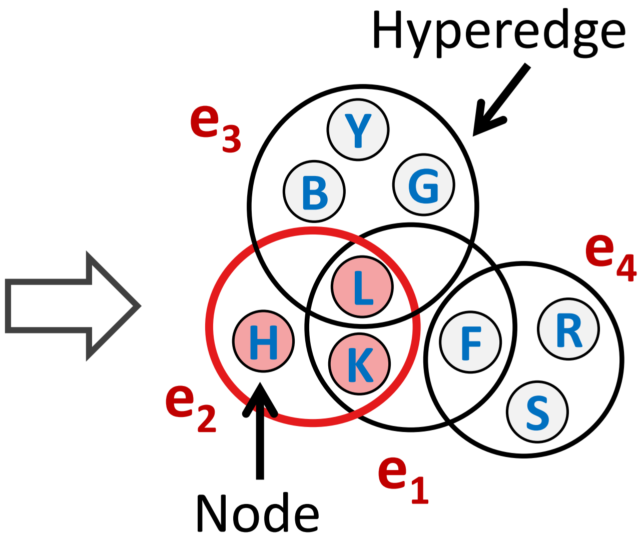

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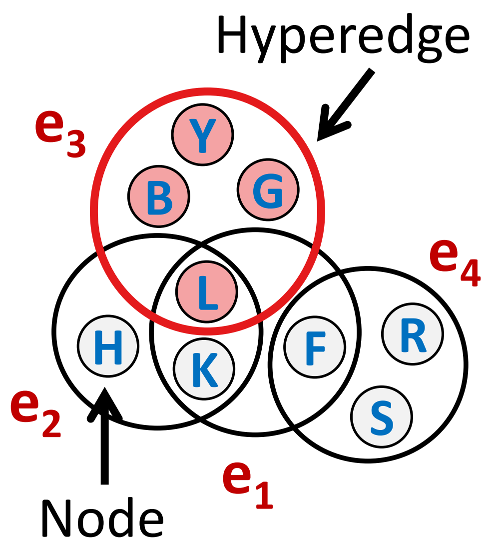

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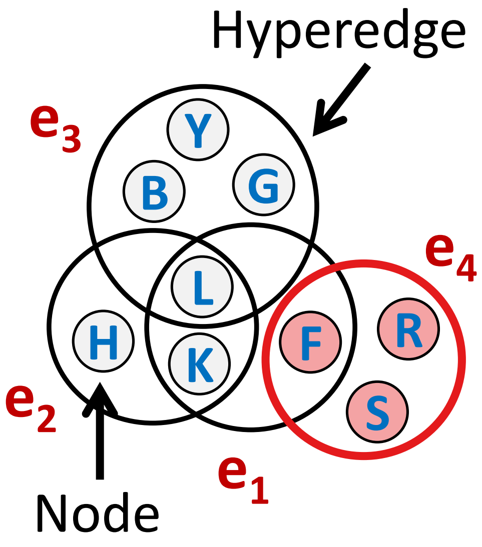
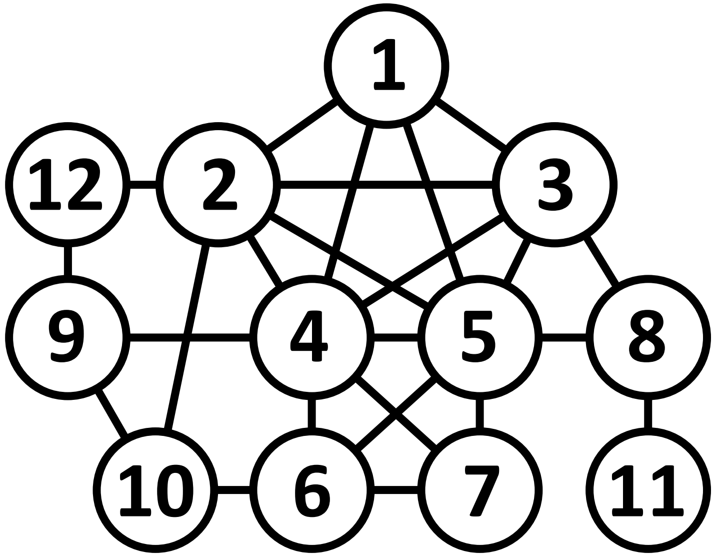


# Roadmap

- **Concepts & Algorithms <<**
- Observations
- Applications
- Conclusions

# Important and useful $\boldsymbol{k}$-cores

- The $\boldsymbol{k}$**-core** $C_k(G)$ of a graph $G$ is the **maximum subgraph** of $G$ such that each node in $C_k(G)$ is **incident** to $\geq k$ edges
  - In (pairwise) graphs, each edge $e = (v_1, v_2)$ consists of a pair of nodes, $v_1$ and $v_2$, and we say both $v_1$ and $v_2$ are incident to this edge $e$
  - The number of edges that a node $v$ is incident to is also called the **degree** of $v$
  - It is maximum in the sense that **no node or edge can be added** into the $k$-core while still satisfying the conditions



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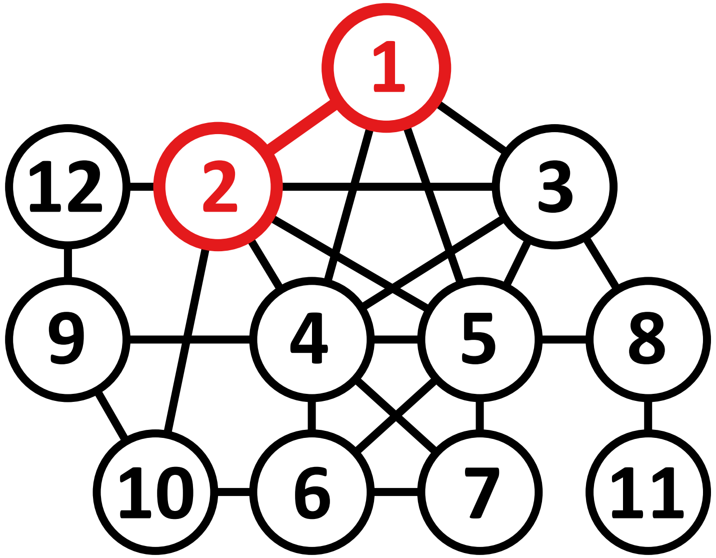

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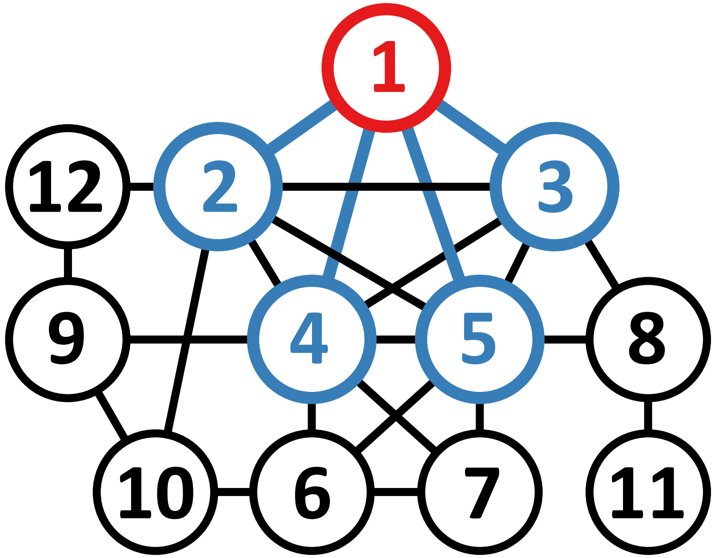

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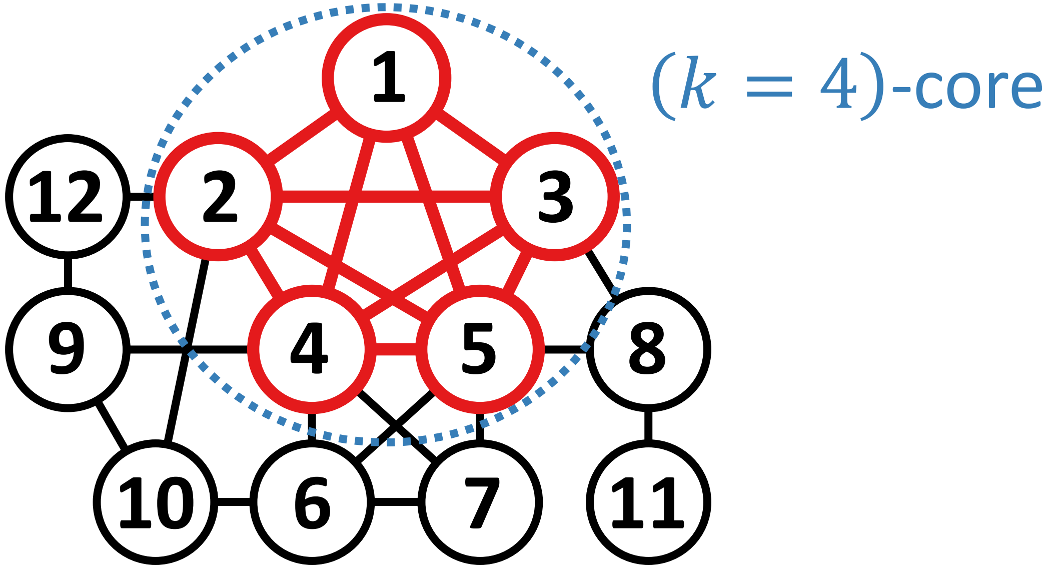
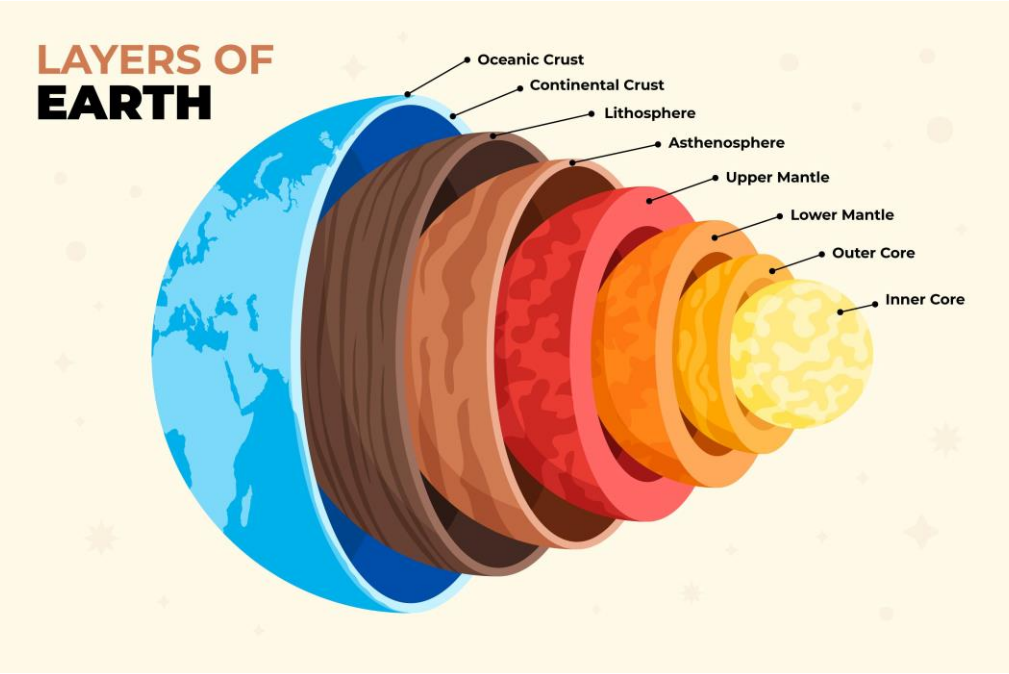


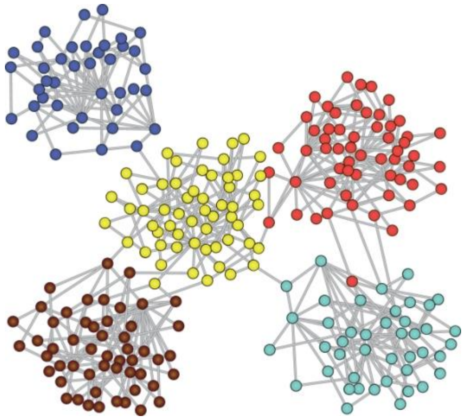
# Important and useful $\boldsymbol{k}$-cores

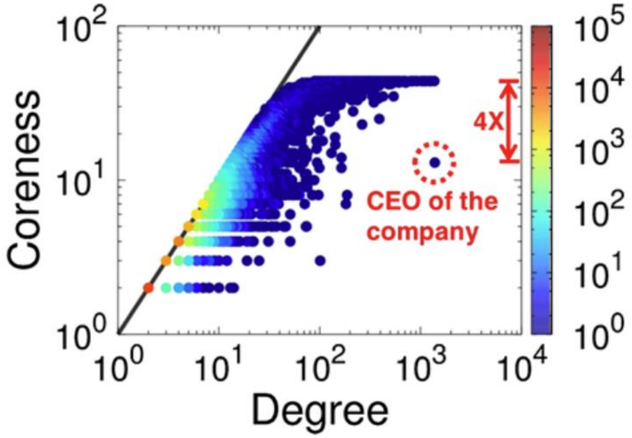
- The $\boldsymbol{k}$**-core** $C_k(G)$ of a graph $G$ is the **maximum subgraph** of $G$ such that each node in $C_k(G)$ is **incident** to $\geq k$ edges
  - We can obtain the $k$-core by keeping removing nodes violating the conditions
  - Whenever a node is removed, **all its incident edges are removed** too

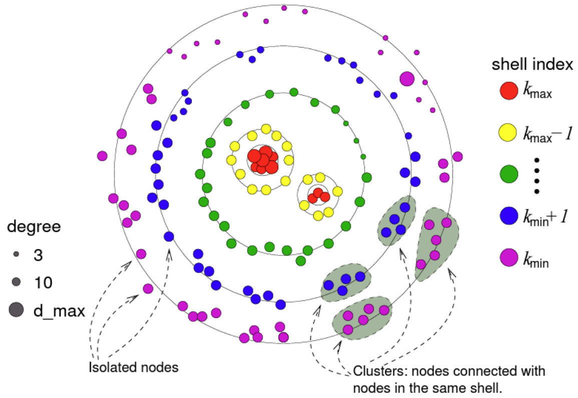


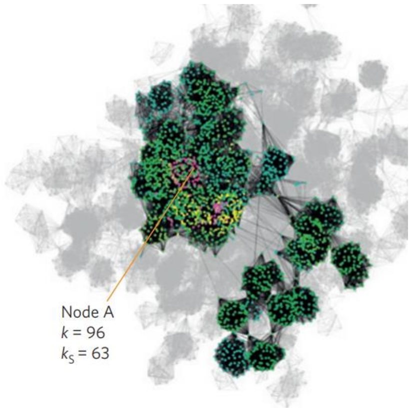

# Important and useful $\boldsymbol{k}$-cores

- The **$\boldsymbol{k}$-core** $C_k(G)$ of a graph $G$ is the **maximum subgraph** of $G$ such that each node in $C_k(G)$ is **incident** to $\geq k$ edges
- **Applications:** community detection, anomaly detection, network visualization, important-node identification, …





# Hypercores: $\boldsymbol{k}$-cores in hypergraphs

- **Q:** How can we generalize $k$-cores in hypergraphs?
- The ***k*-core** $C_k(G)$ of a graph $G$ is the maximum **subgraph** of $G$ such that each node in $C_k(G)$ is incident to $\geq k$ **edges**
- The ***k*-hypercore** $C_k(H)$ of a hypergraph $H$ is the max. **subhypergraph** of $H$ such that each node in $C_k(H)$ is incident to $\geq k$ **hyperedges**
- **Q:** What kind of subhypergraphs should we allow?

# Hypercores: $\boldsymbol{k}$-cores in hypergraphs

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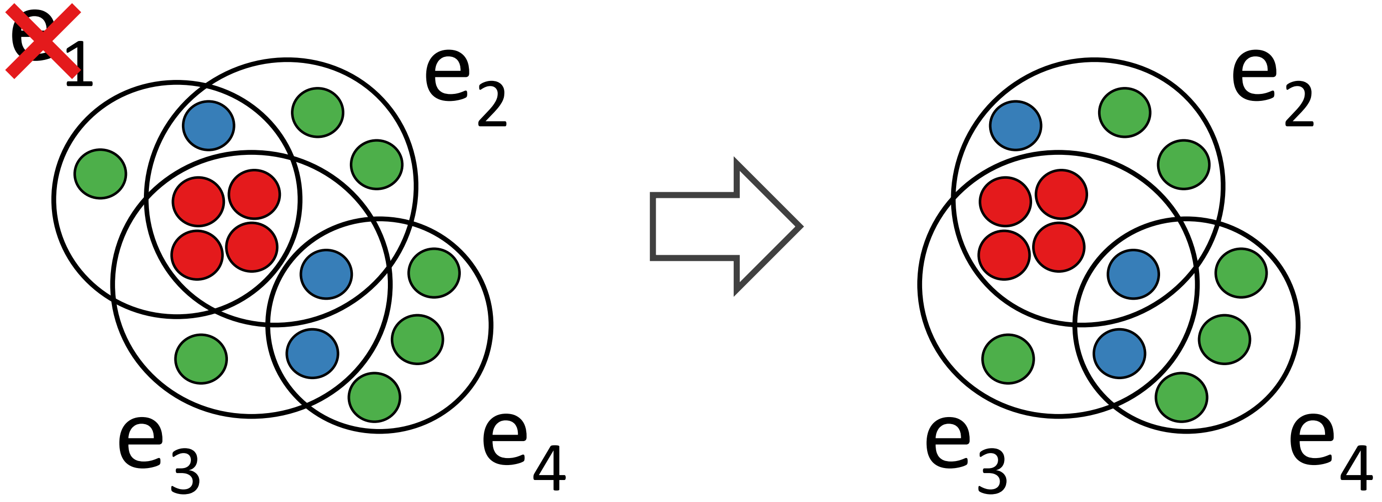
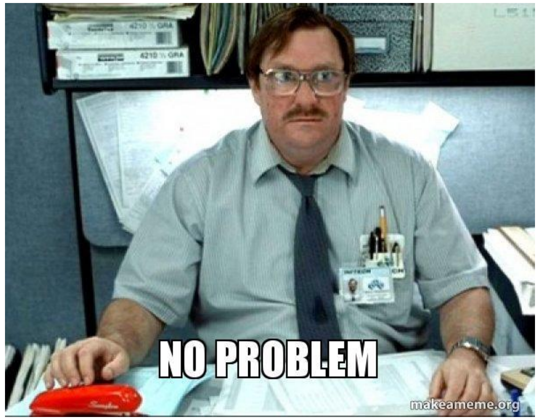
- **Q:** What kind of **subhypergraphs** should be allowed in $k$-hypercores?
  - Simply removing **some of the hyperedges**?



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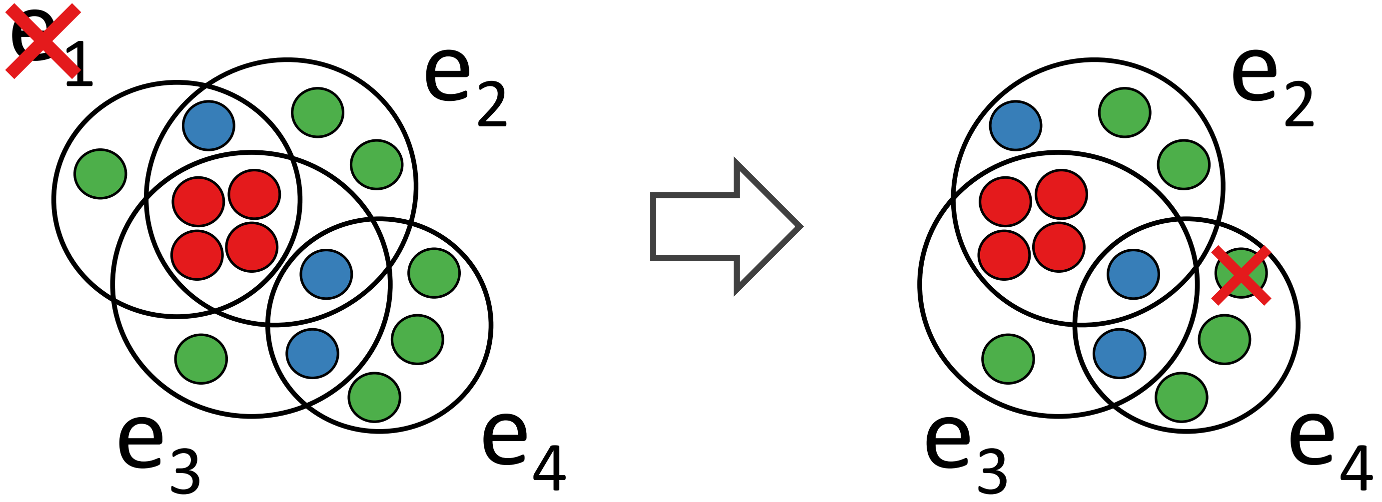
- **Q:** What kind of **subhypergraphs** should be allowed in $k$-hypercores?
  - What if we remove **some of the nodes from a hyperedge**?

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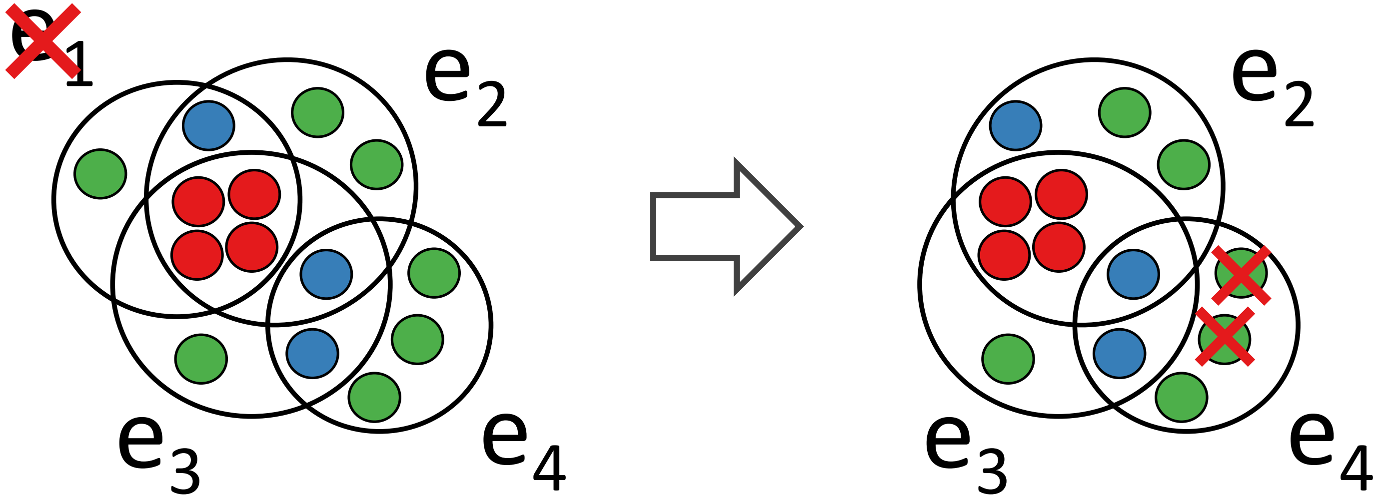
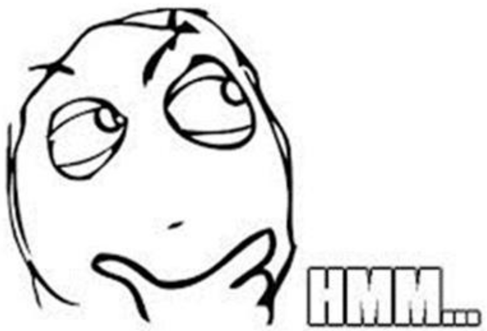
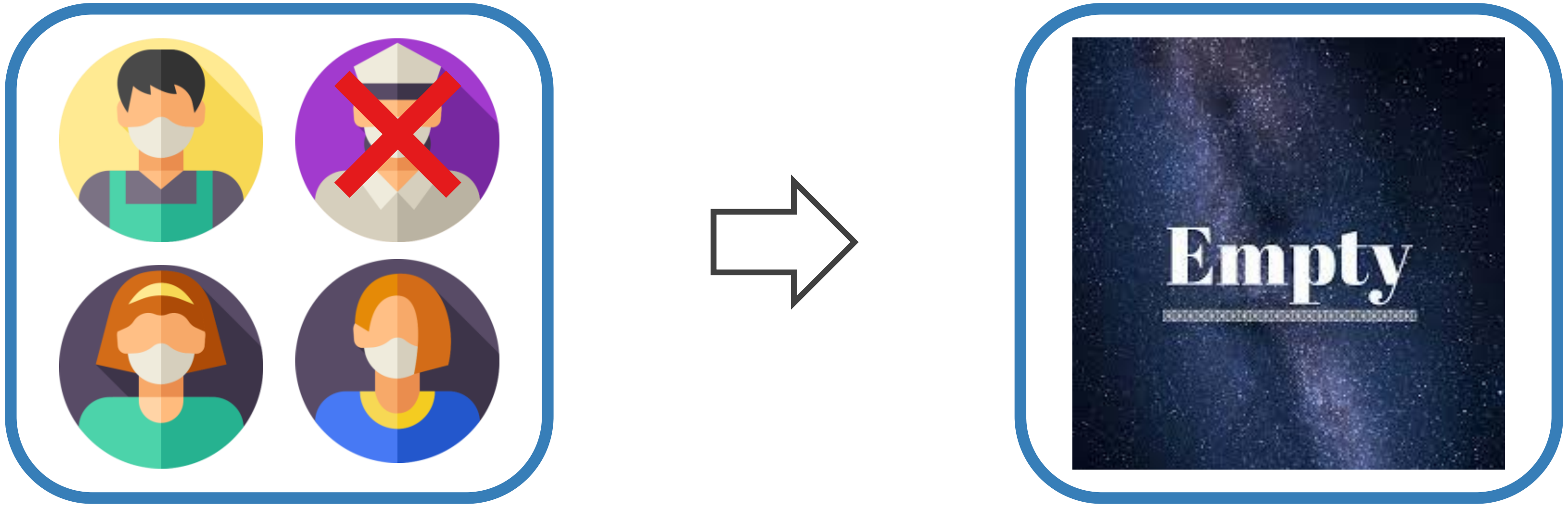
- **Q:** What kind of **subhypergraphs** should be allowed in $k$-hypercores?
  - **How many** nodes are allowed to leave a hyperedge?



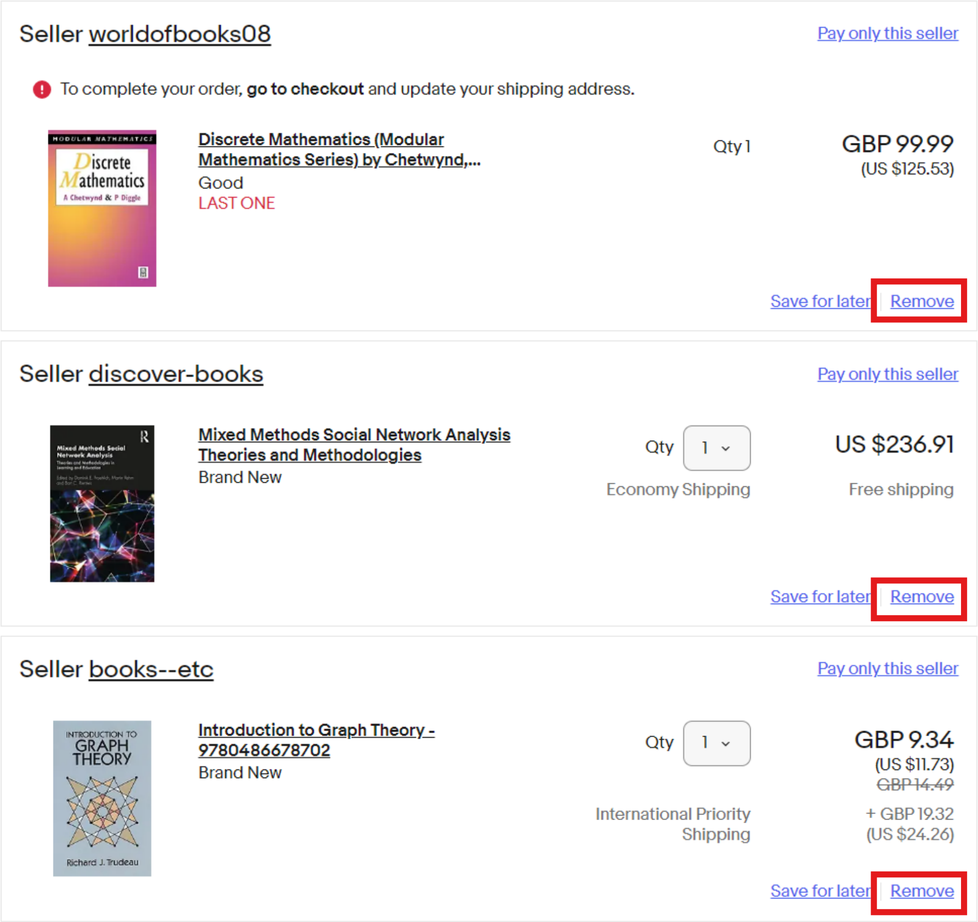
# A Naïve Definition with Fragile Hyperedges

- Hyperedges are **fragile**, with **every** member **indispensable**
- A hyperedge is kept only if **all** the constituent nodes remain
- But this is **NOT** realistic!



# Real-World Groups are Not Fragile

- Many real-world groups still remain valid even if some members leave

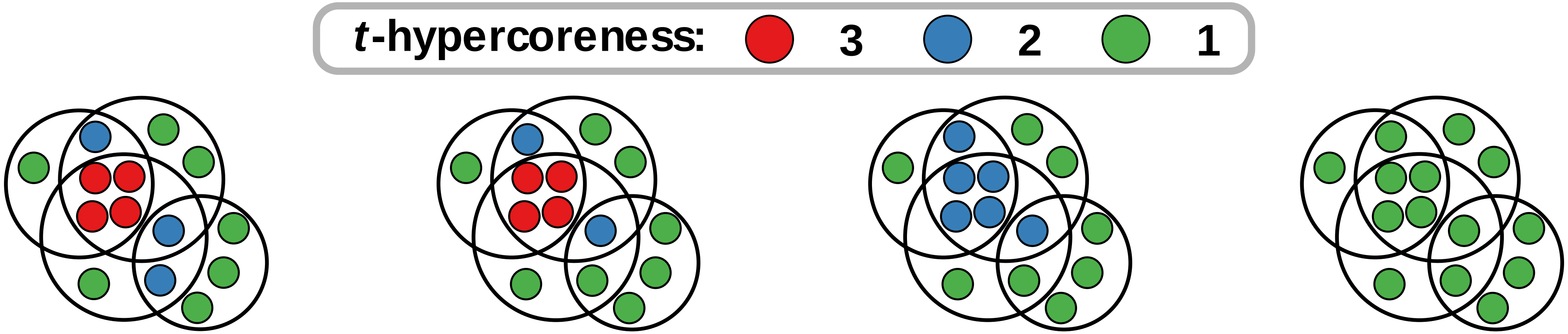



# Definition: $(\boldsymbol{k}, \boldsymbol{t})$-Hypercores

- Given a hypergraph $H = (V, E)$, a positive integer $k$, and $\boldsymbol{t} \in [\mathbf{0}, \mathbf{1}]$
- The $(\boldsymbol{k}, \boldsymbol{t})$**-hypercore** $C_{k,t}(H)$ of $H$, is the maximum subhypergraph of $H$ such that
  - Each **node** in $C_{k,t}(H)$ is incident to $\geq k$ hyperedges
  - Each **hyperedge** in $C_{k,t}(H)$ contains $\geq t$ proportion of the original constituent nodes (and at least two nodes)
- **Example:** $t = 3/4$

# Related Concepts: $\boldsymbol{t}$-Hypercoreness & $\boldsymbol{k}$-Fraction

- The **$\boldsymbol{t}$-hypercoreness** $c_t(v)$ of a node $v$ is the **maximum** $k^*$ such that $v$ is in the $(k^*, t)$-hypercore
- The **$\boldsymbol{k}$-fraction** $f_k(v)$ of a node $v$ is the maximum $t^*$ such that $v$ is in the $(k, t^*)$-hypercore
- **Example:** Different $(k, t)$-hypercore structures with different $t$ values




$t \leq 2/5$ $\quad$ $2/5 \leq t \leq 4/7$ $\quad$ $4/7 \leq t \leq 5/7$ $\quad$ $t > 5/7$

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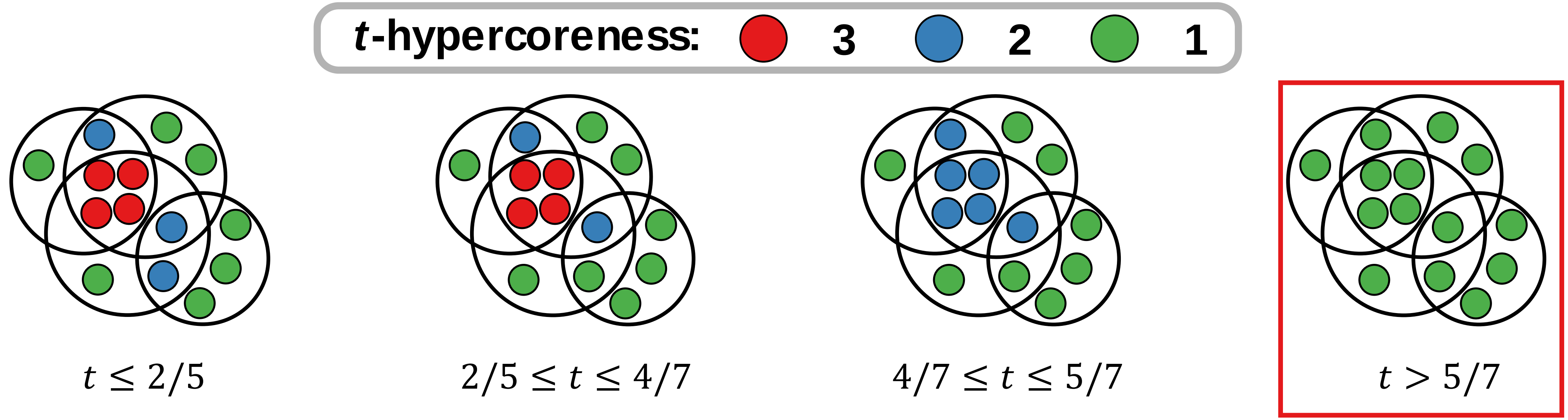
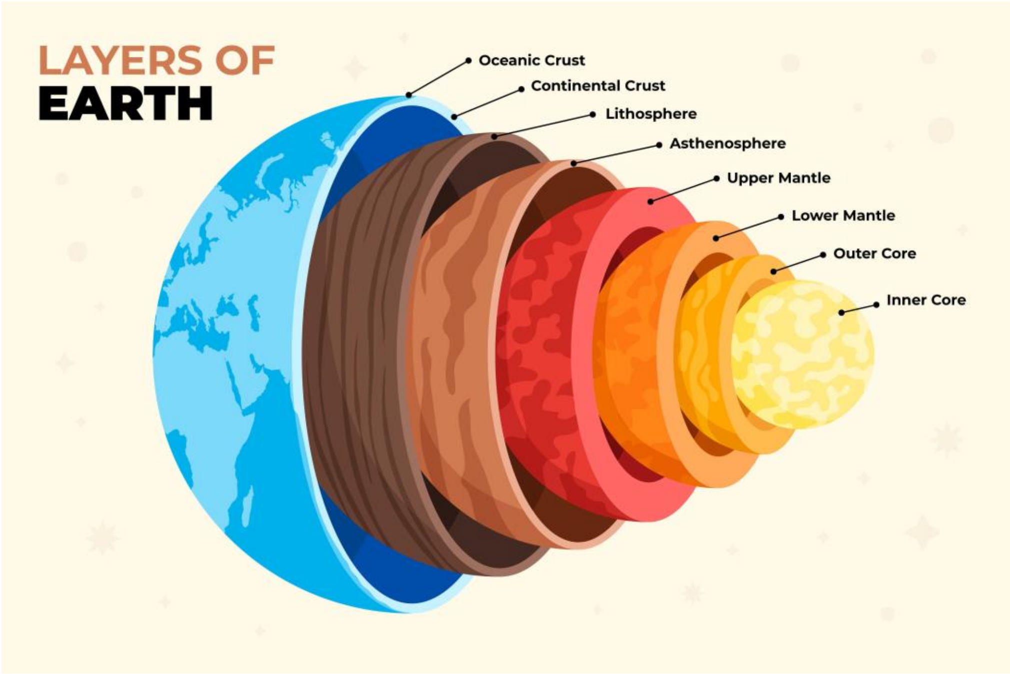


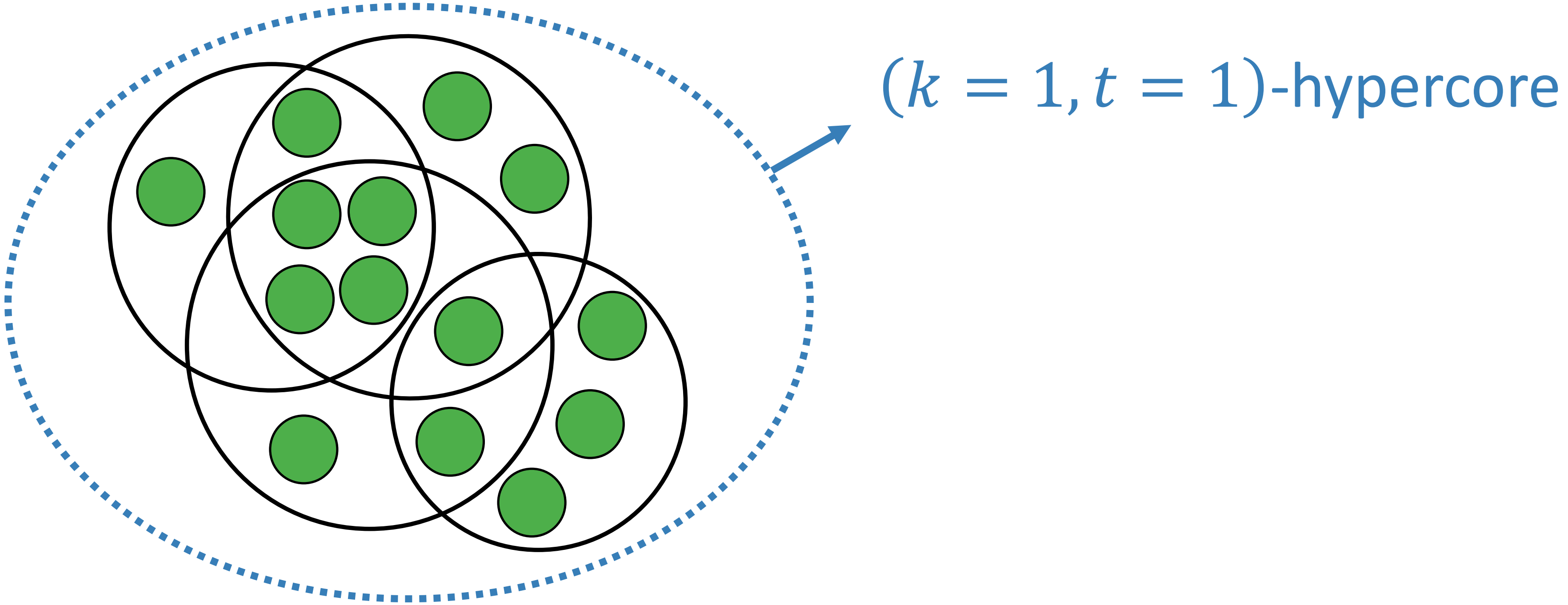
# Computation Algorithms: Find the Core

- **Peeling:** We repeatedly remove nodes and hyperedges violating the conditions, until the conditions are satisfied
- **Time complexity: Linear** to the total size of the input hypergraph
  - Specifically, $O(\sum_{e \in E} |e|)$




# Computation Algorithms: Example ($\boldsymbol{t = 1}$)

- **Example:** how we obtain a $(k, t)$-hypercore



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- **Example:** how we obtain a $(k, t)$-hypercore
  - Each **node** should be incident to $\geq k$ hyperedges
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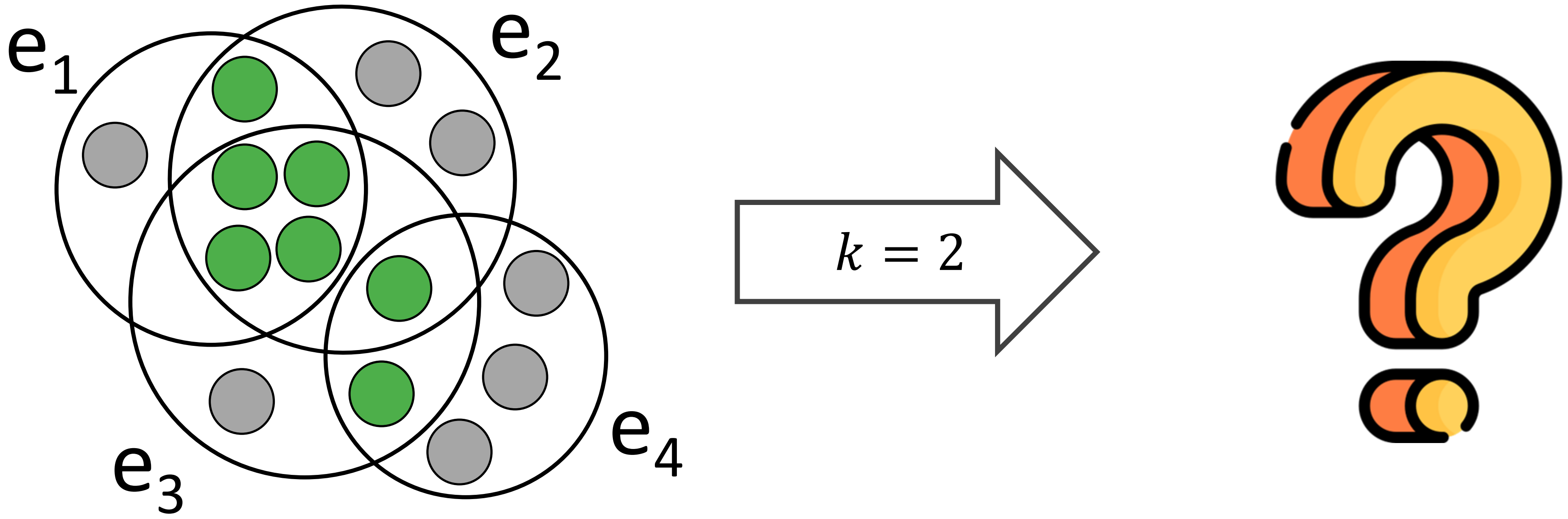

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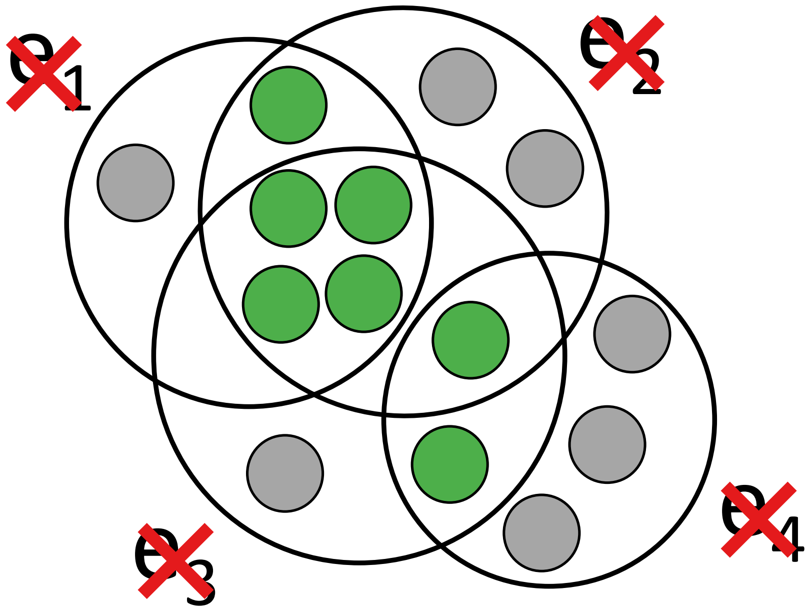
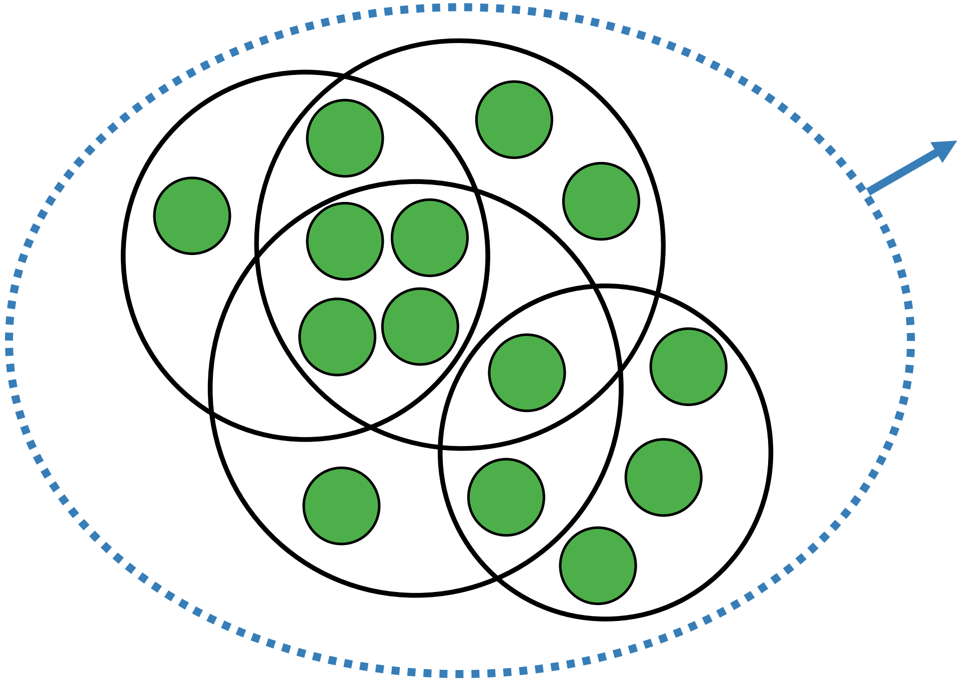



$k = 2$

# Computation Algorithms: Example ($\boldsymbol{t} = \mathbf{1/2}$)

- **Example:** how we obtain a $(k, t)$-hypercore



$(k = 1, t = 1/2)$-hypercore

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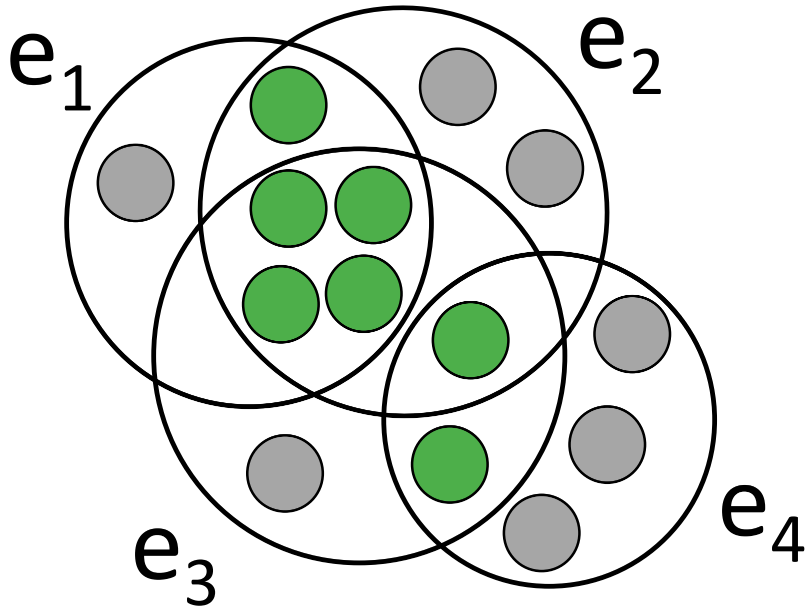


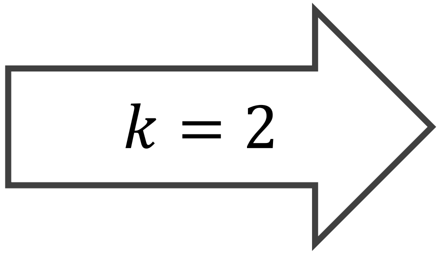


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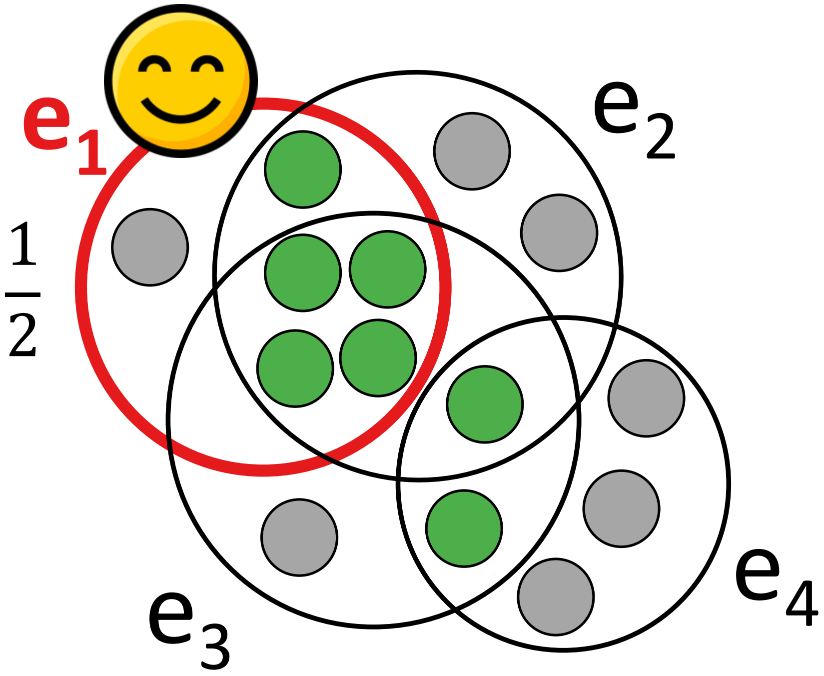
$$\frac{5}{6} > \frac{1}{2}$$

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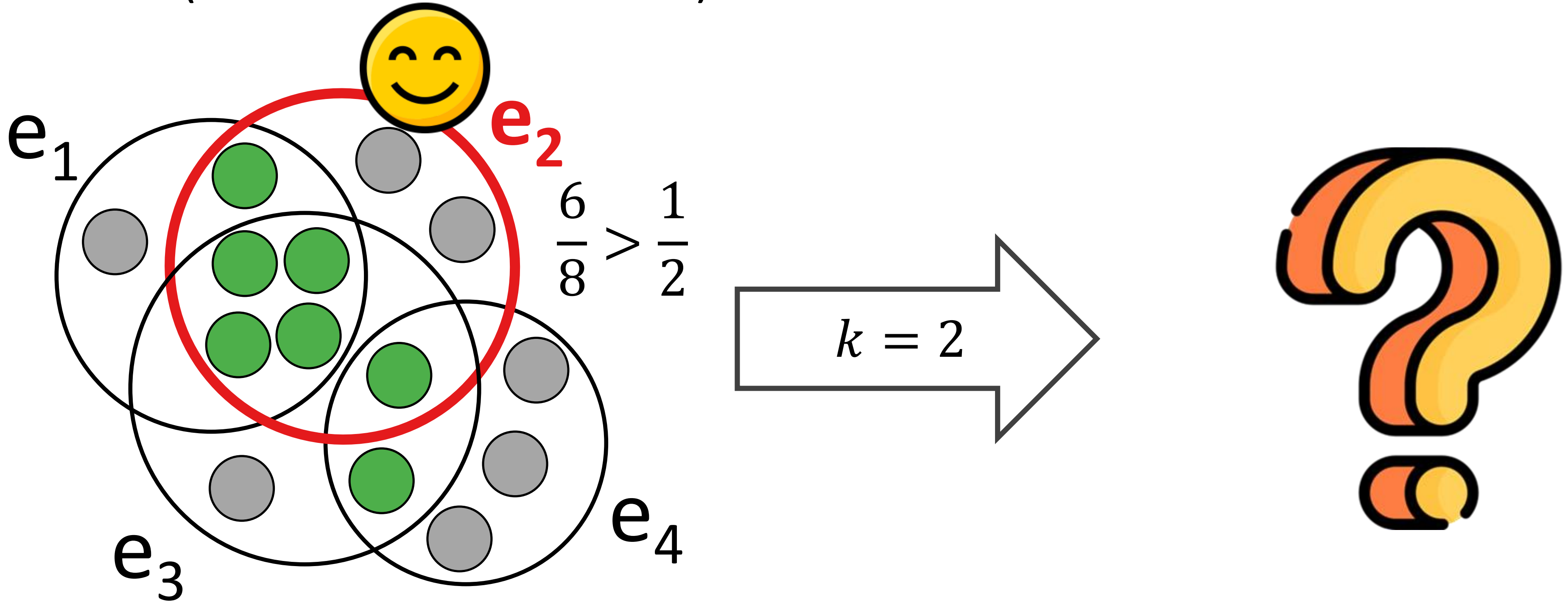

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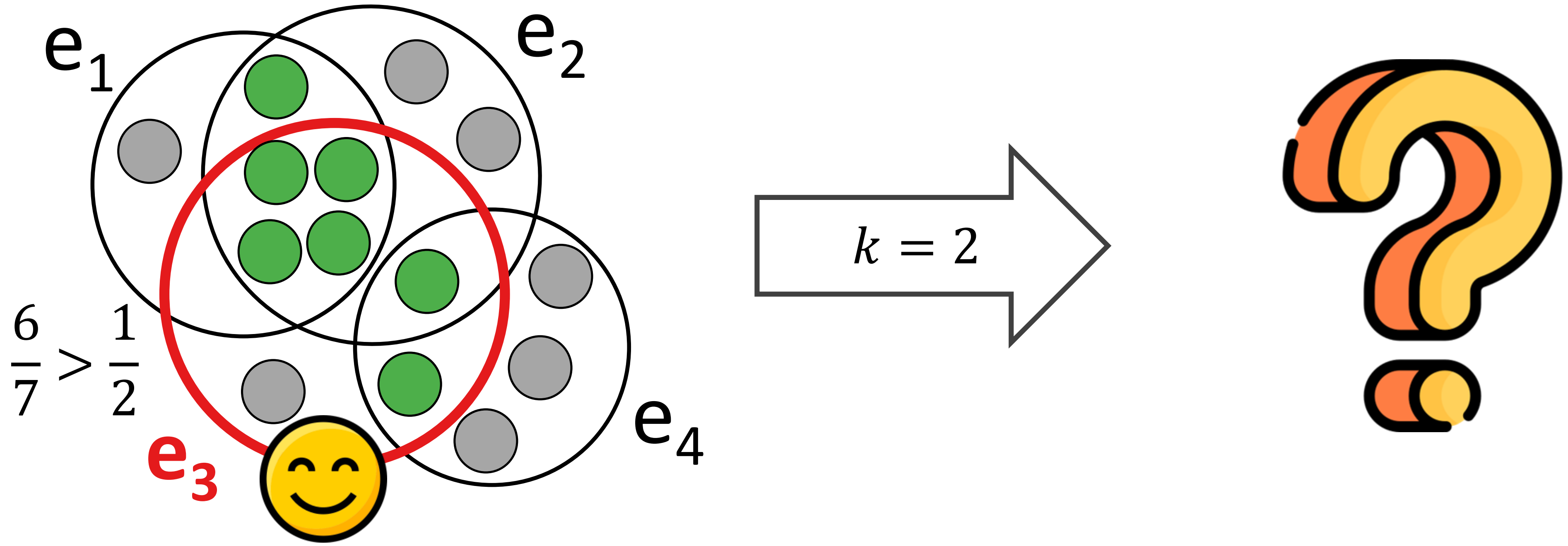

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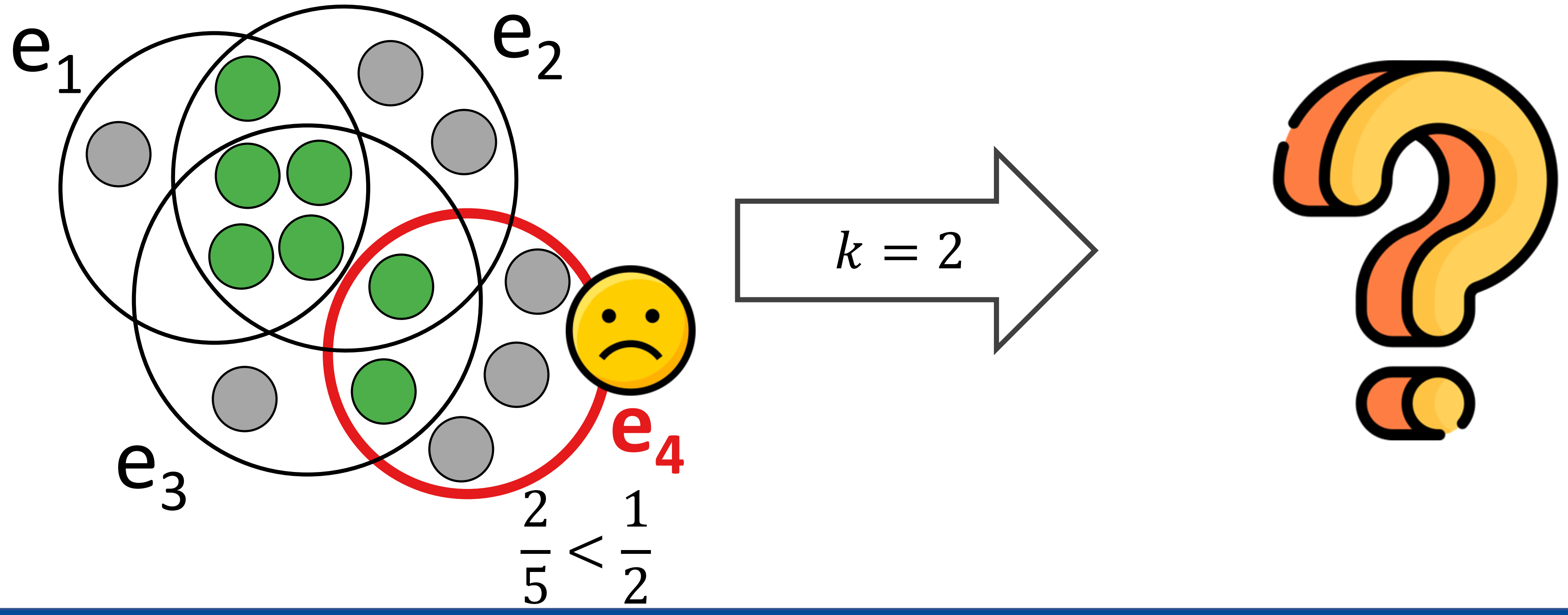

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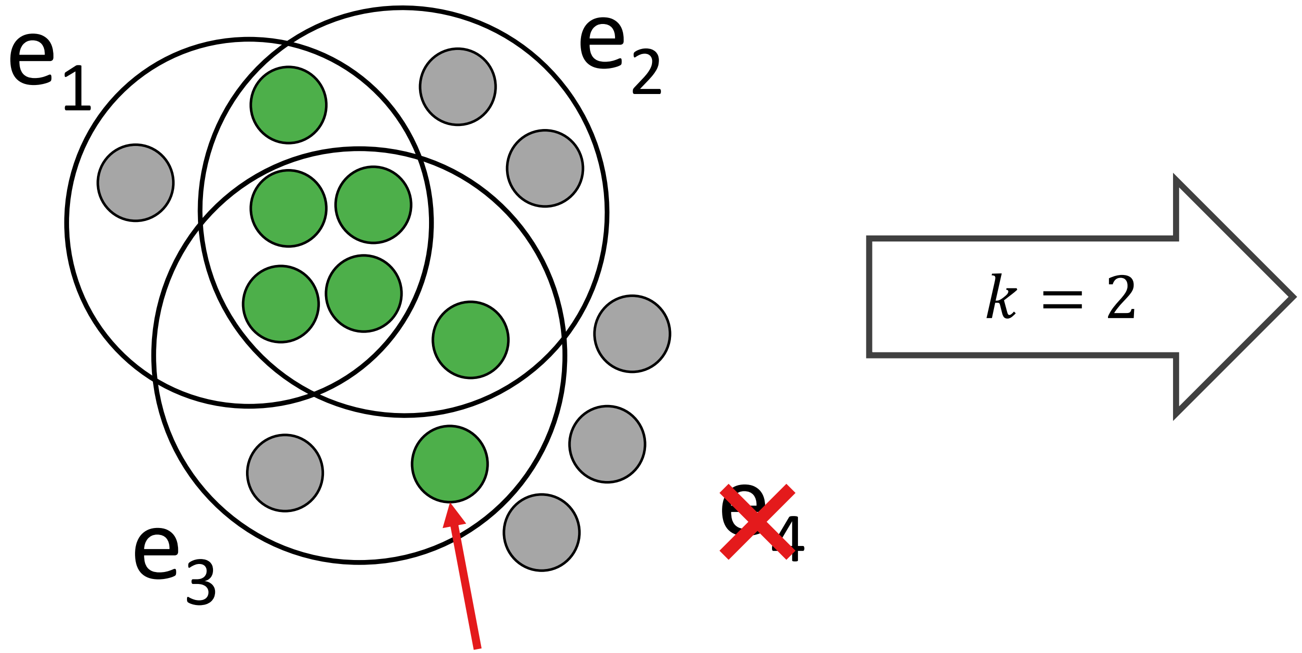


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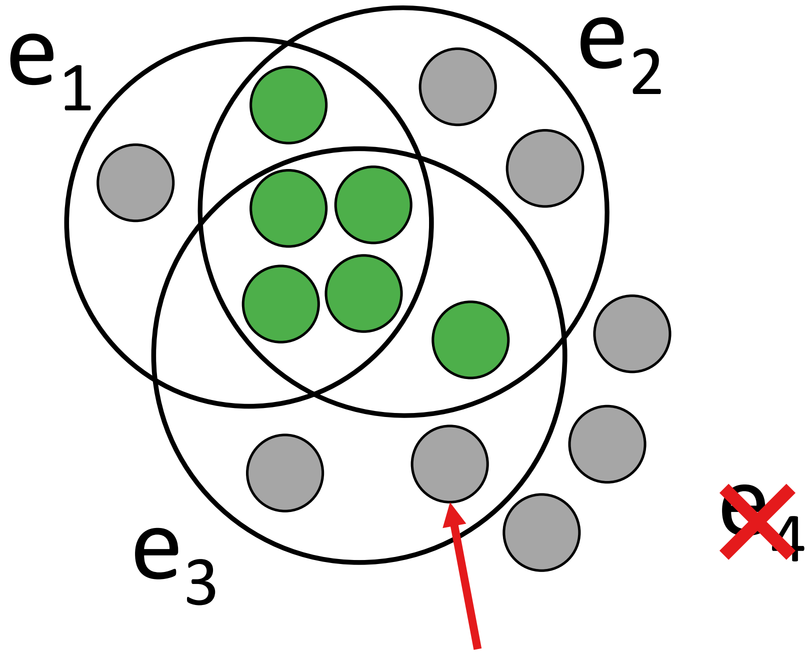


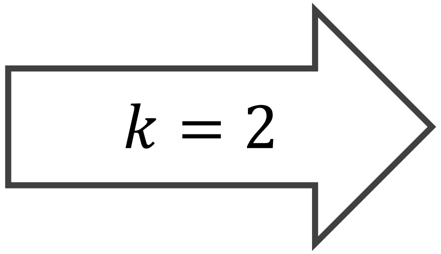


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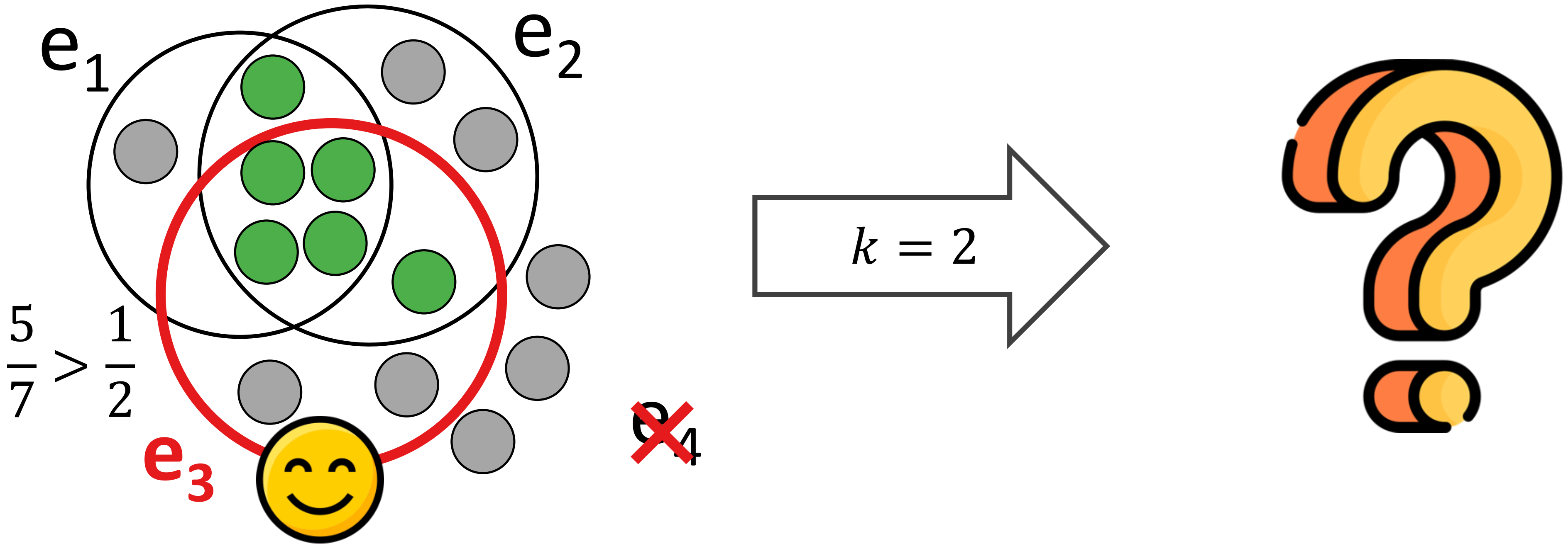

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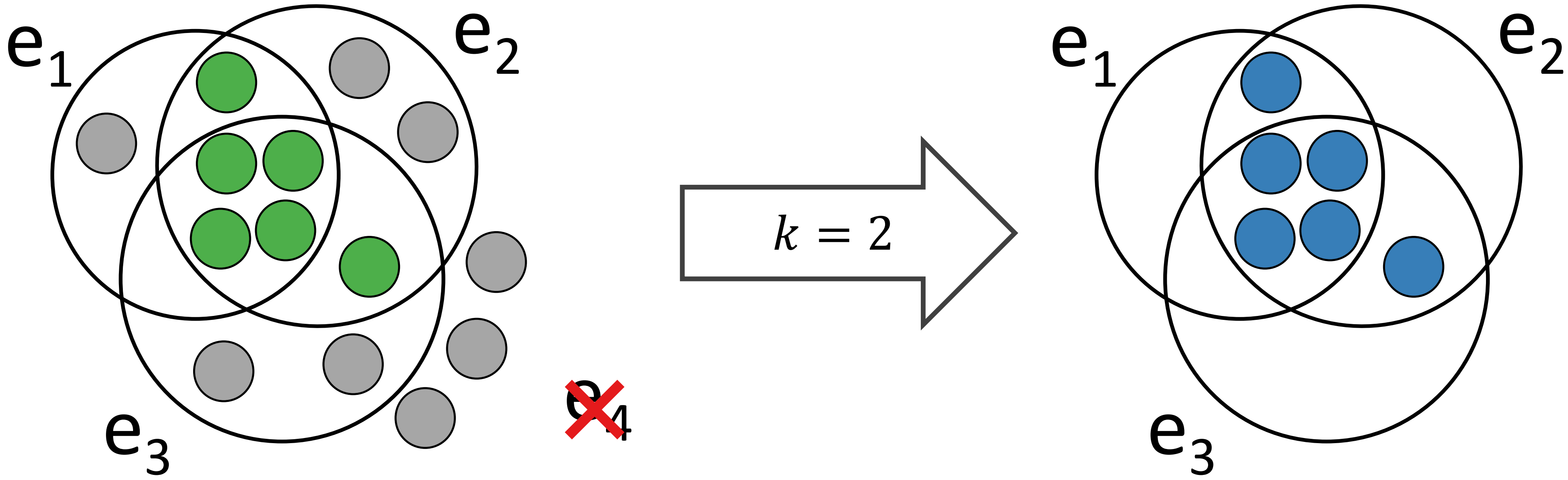
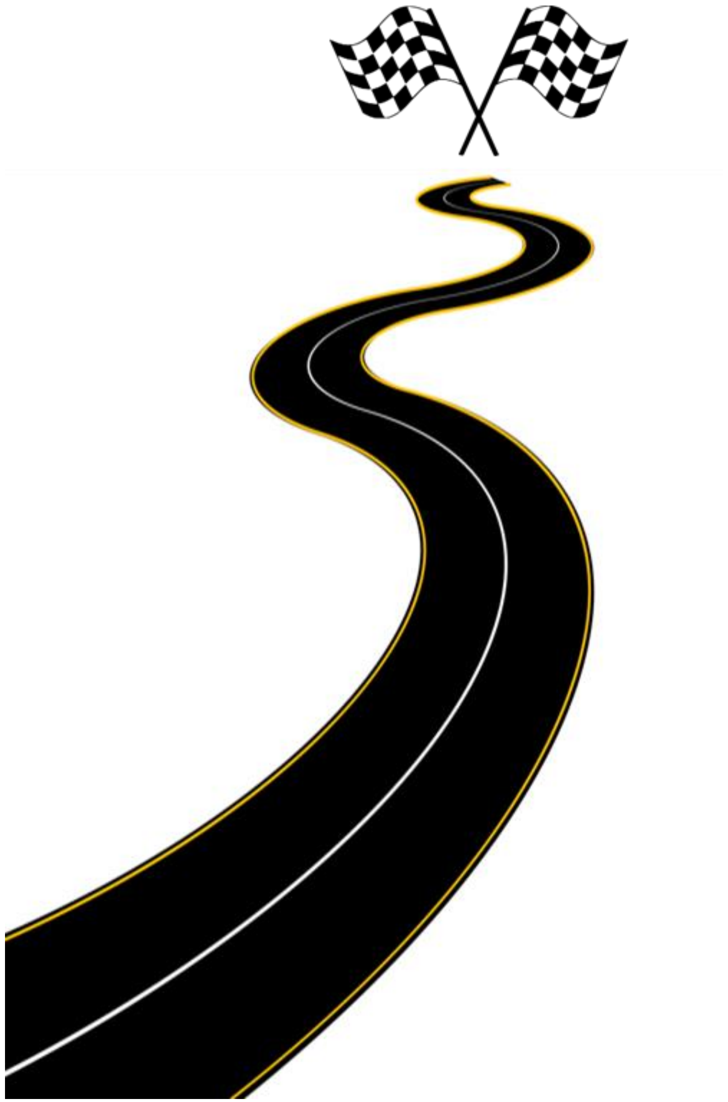


# Roadmap

- Concepts & Algorithms
- **Observations <<**
- Applications
- Conclusions



# Real-World Hypergraph Datasets

- **Datasets:** 14 real-world hypergraphs from 6 different domains
  - # nodes: 143 – 2.3M
  - # hyperedges: 1,047 – 8.6M
- **Source:** https://www.cs.cornell.edu/~arb/data/

| Dataset | $\lvert V \rvert$ | $\lvert E \rvert$ | max./avg. $d(v)$ | max./avg. $\lvert e \rvert$ |
|---|---|---|---|---|
| coauth-DBLP | 1,831,126 | 2,169,663 | 846 / 4.06 | 25 / 3.42 |
| coauth-Geology | 1,087,111 | 908,516 | 716 / 3.21 | 25 / 3.84 |
| NDC-classes | 1,149 | 1,047 | 221 / 5.57 | 24 / 6.11 |
| NDC-substances | 3,438 | 6,264 | 578 / 14.51 | 25 / 7.96 |
| contact-high | 327 | 7,818 | 148 / 55.63 | 5 / 2.33 |
| contact-primary | 242 | 12,704 | 261 / 126.98 | 5 / 2.42 |
| email-Enron | 143 | 1,457 | 116 / 31.43 | 18 / 3.09 |
| email-Eu | 979 | 24,399 | 910 / 86.93 | 25 / 3.49 |
| tags-ubuntu | 3,021 | 145,053 | 12,930 / 164.56 | 5 / 3.43 |
| tags-math | 1,627 | 169,259 | 13,949 / 363.80 | 5 / 3.50 |
| tags-SO | 49,945 | 5,517,054 | 520,468 / 427.77 | 5 / 3.87 |
| threads-ubuntu | 90,054 | 115,987 | 2,170 / 2.97 | 14 / 2.31 |
| threads-math | 153,806 | 535,323 | 11,358 / 9.08 | 21 / 2.61 |
| threads-SO | 2,321,751 | 8,589,420 | 34,925 / 9.75 | 25 / 2.64 |

- **Coauth:** Co-authorship
- **NDC:** National Drug Code
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| Domain | Nodes | Hyperedges |
|---|---|---|
| coauth | Researchers | Publications |
| NDC | Classes/Substances | Drugs |
| contact | People | Communications |
| email | Senders/Receivers | Emails |
| tags | Tags | Questions |
| threads | Users | Threads |

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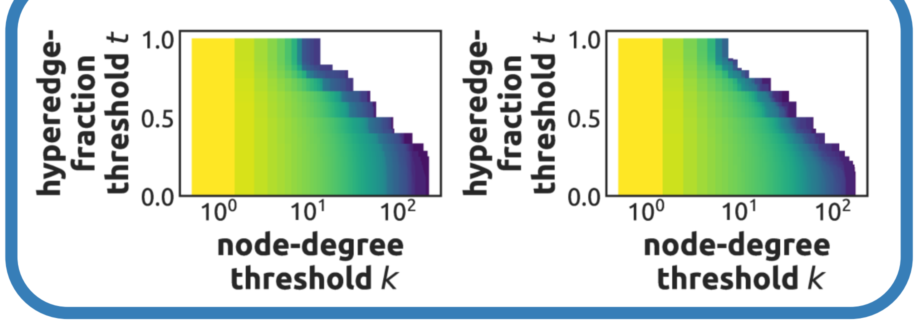
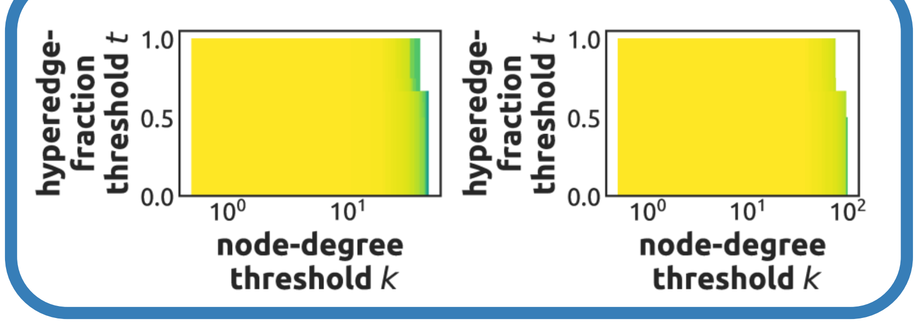
# Observation 1: Domain-Based Patterns of Hypercore Sizes

- **Observation:** Real-world hypergraphs in the same domain have similar patterns of hypercore sizes with different $k$ and $t$ values
- **Color of each pixel:** The normalized size of the $(k, t)$-hypercore

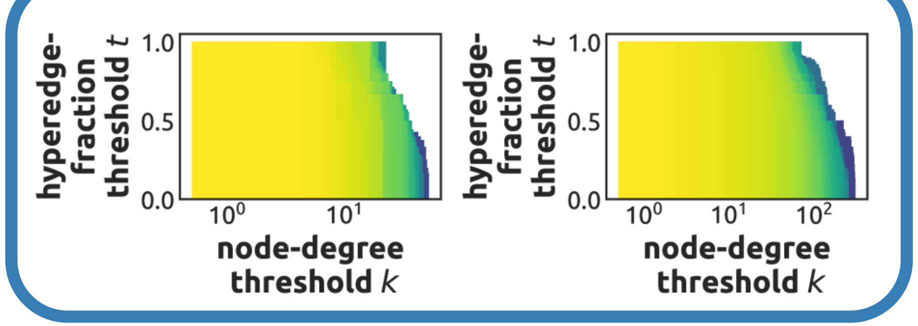
Co-authorship



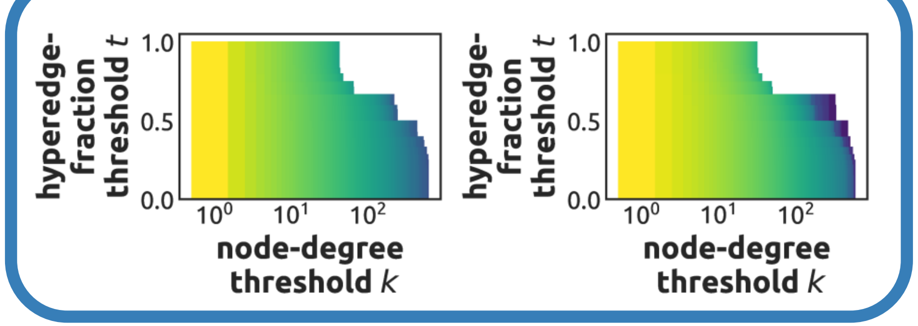

Contact




Email

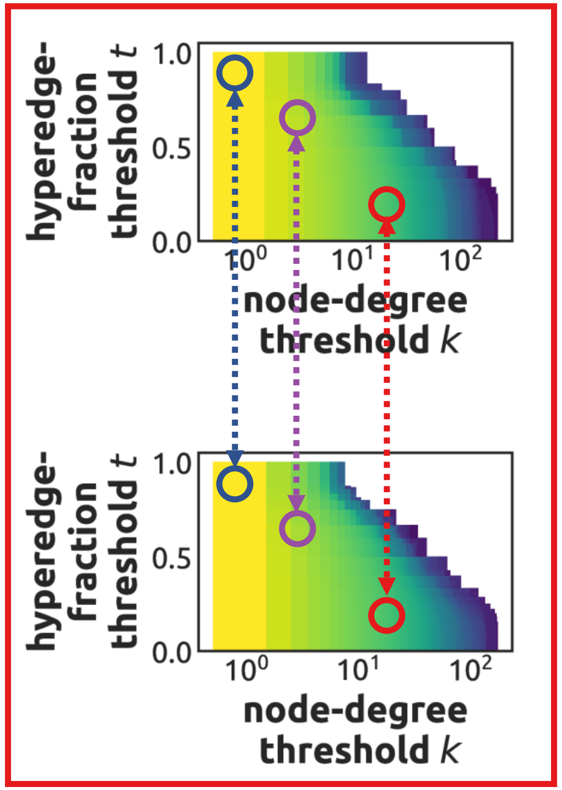


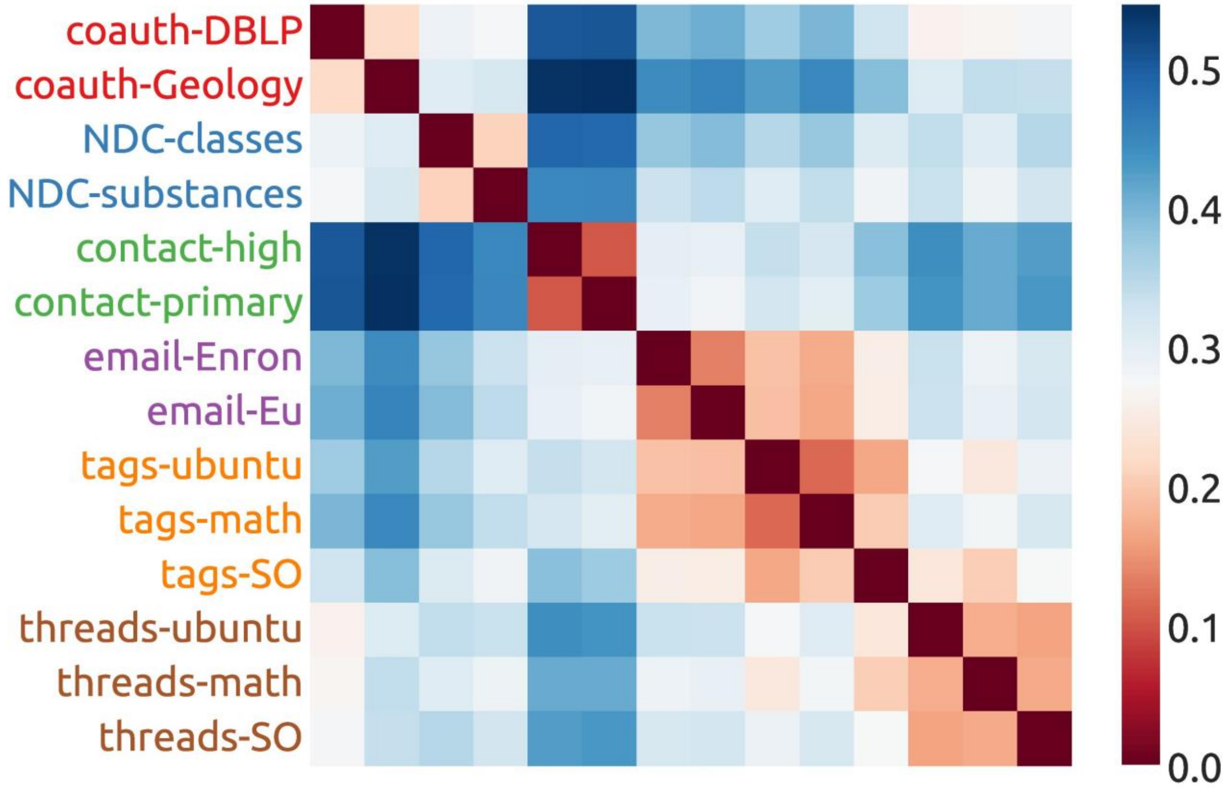

Threads



# Observation 1: Pairwise Distance

- We define a **distance metric** to compare the patterns of hypercore sizes between different hypergrpahs, which can be interpreted as the average **pixel-level difference** between the plots in the previous page



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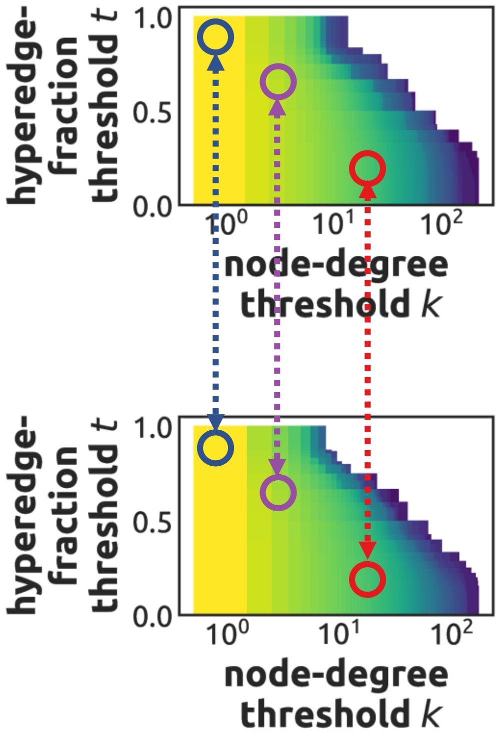
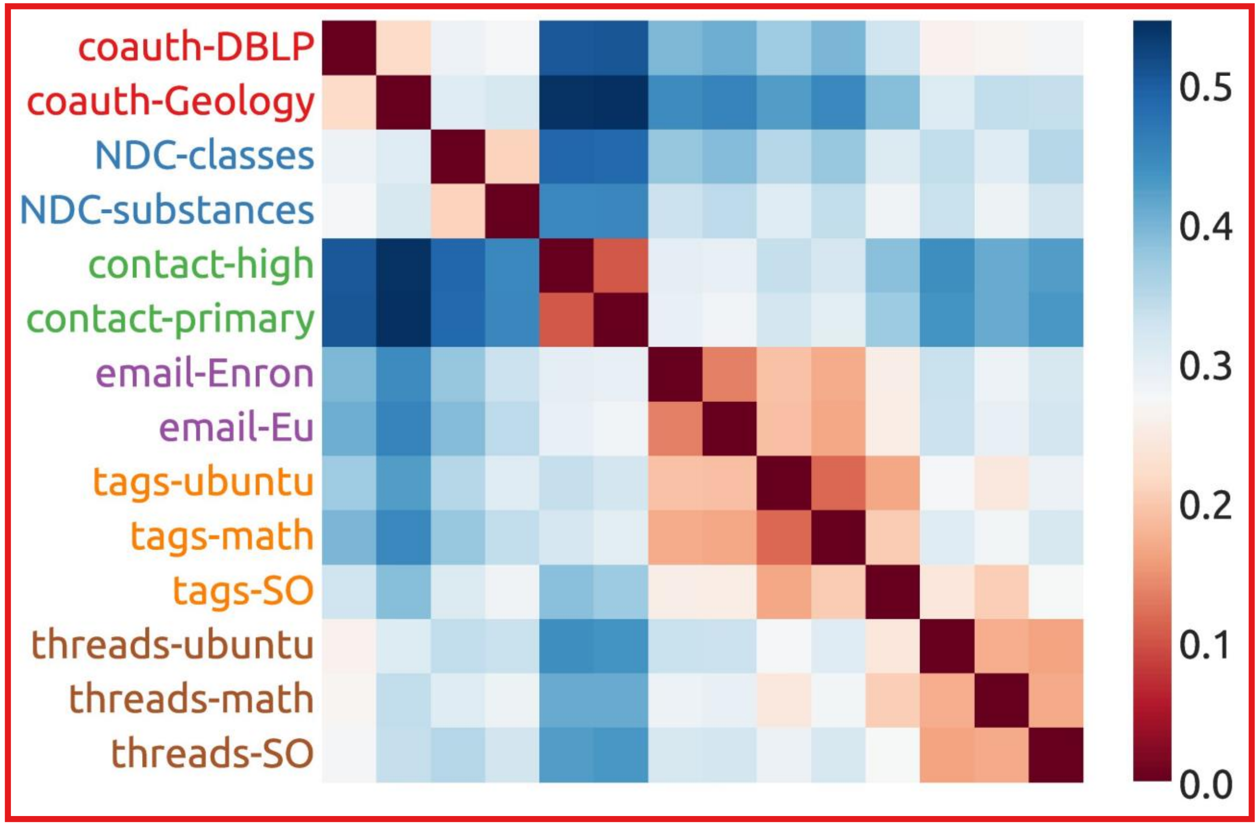
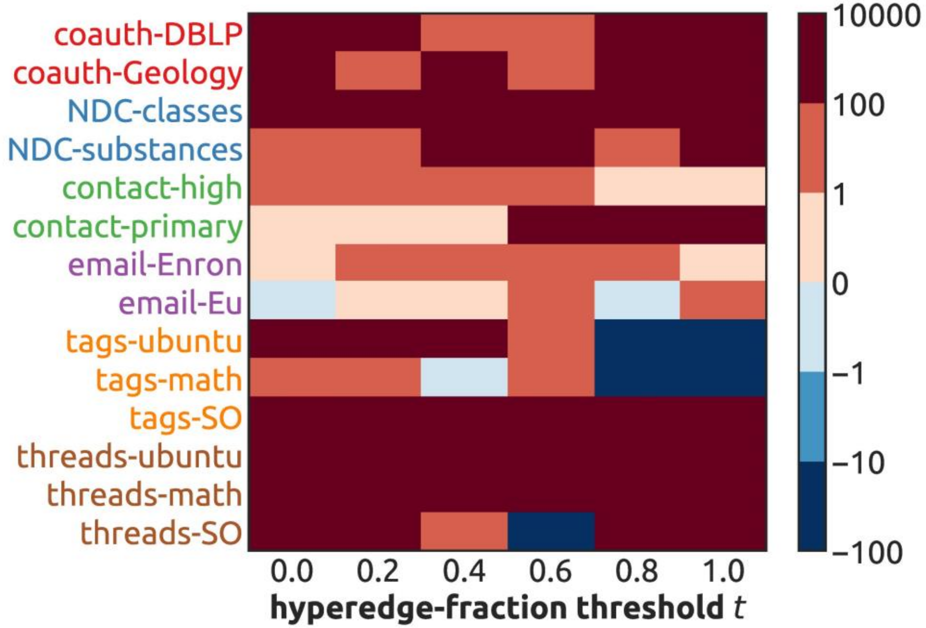
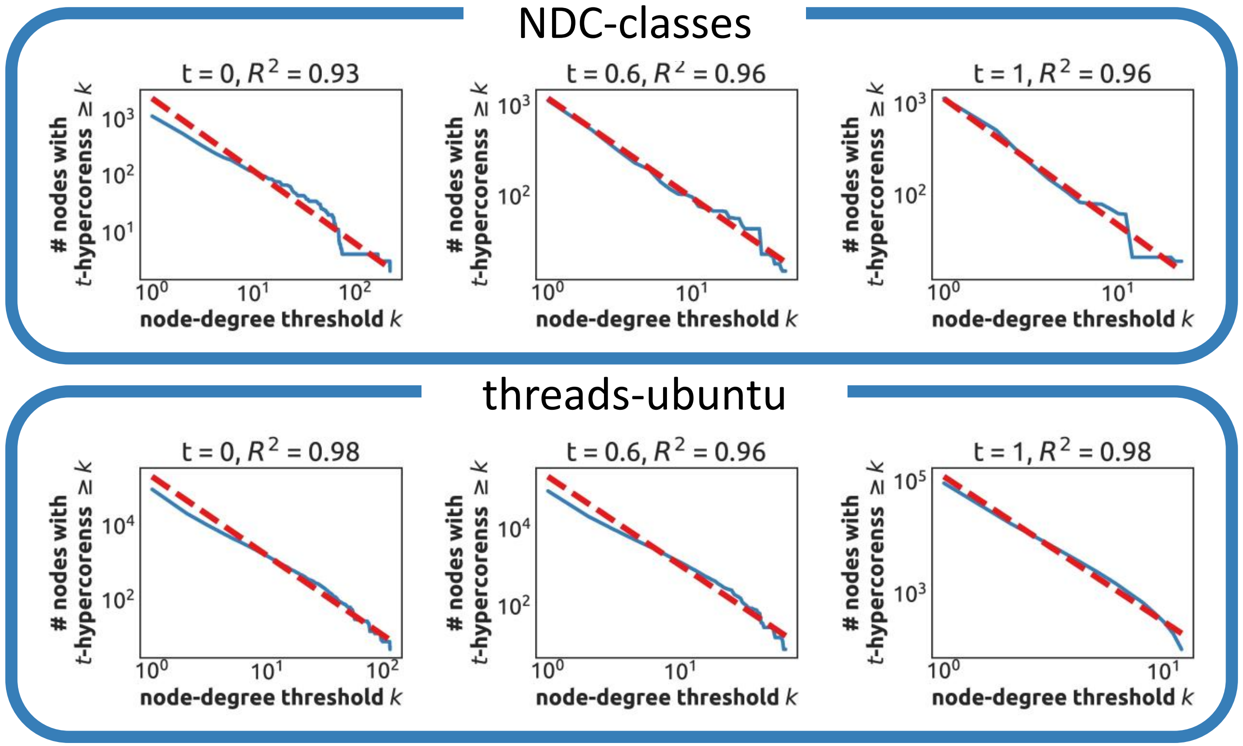


# Observation 2: Heavy-Tailed Hypercoreness Distributions

- **Observation:** In real-world hypergraphs, $t$-hypercoreness follows **heavy-tailed** distributions regardless of t. In some datasets, the $t$-hypercoreness strongly follows a **power law**.
- **Left:** **Red colors** indicates heavy-tailed distributions



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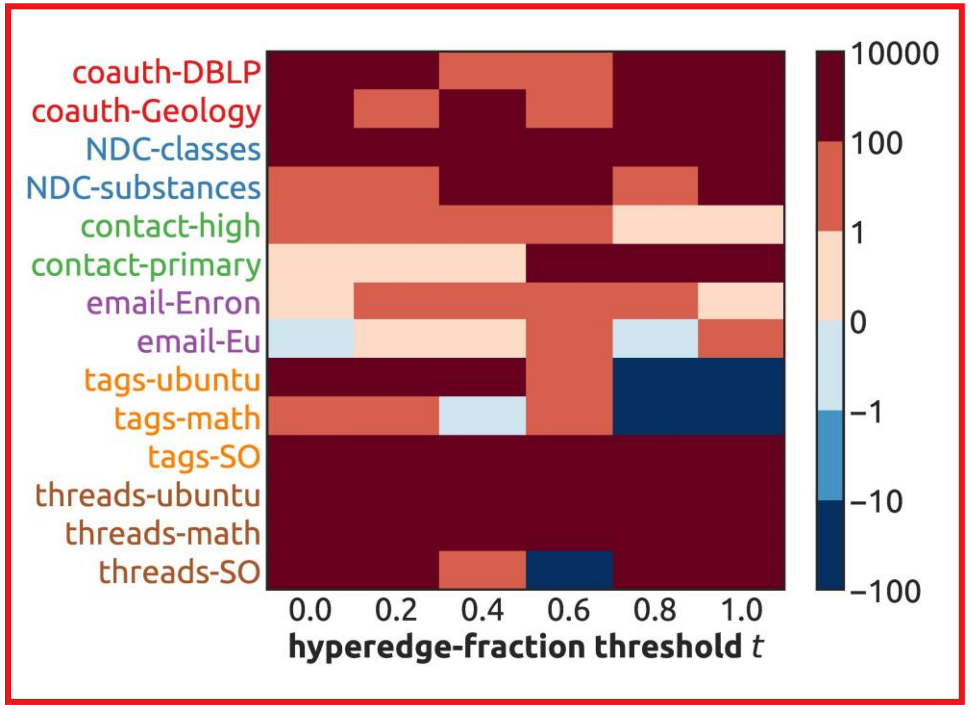


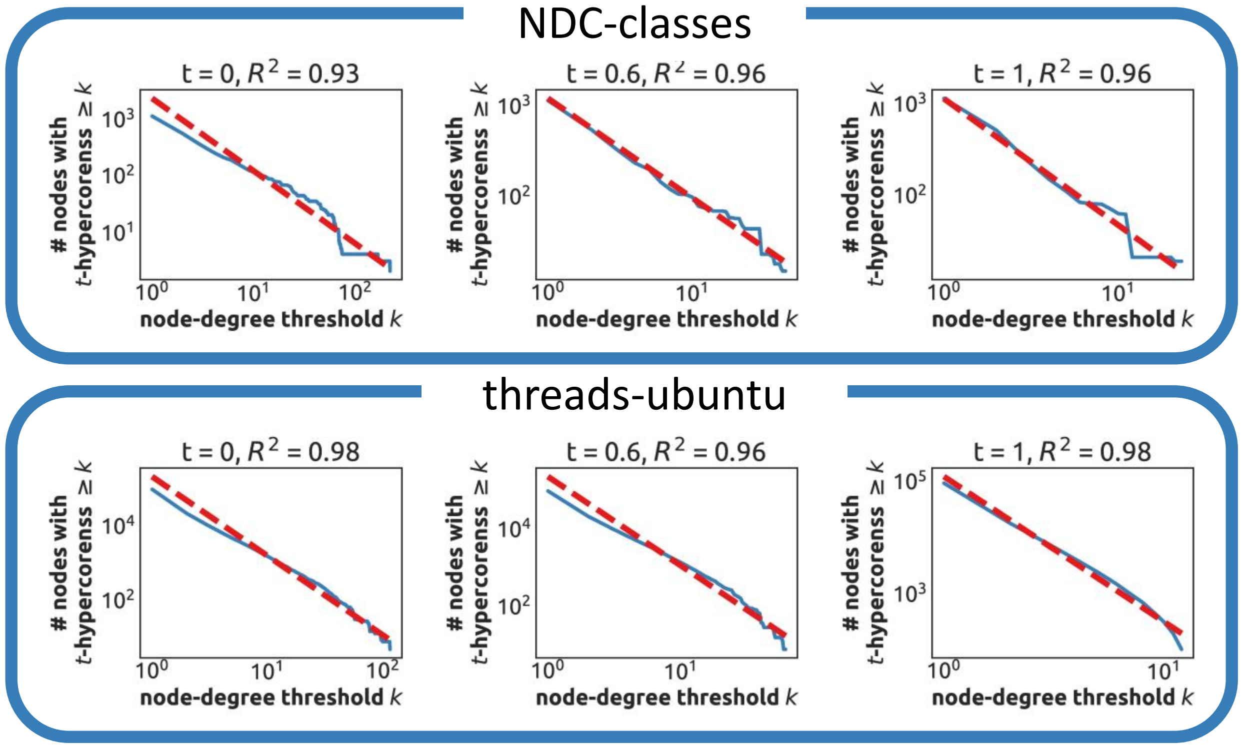

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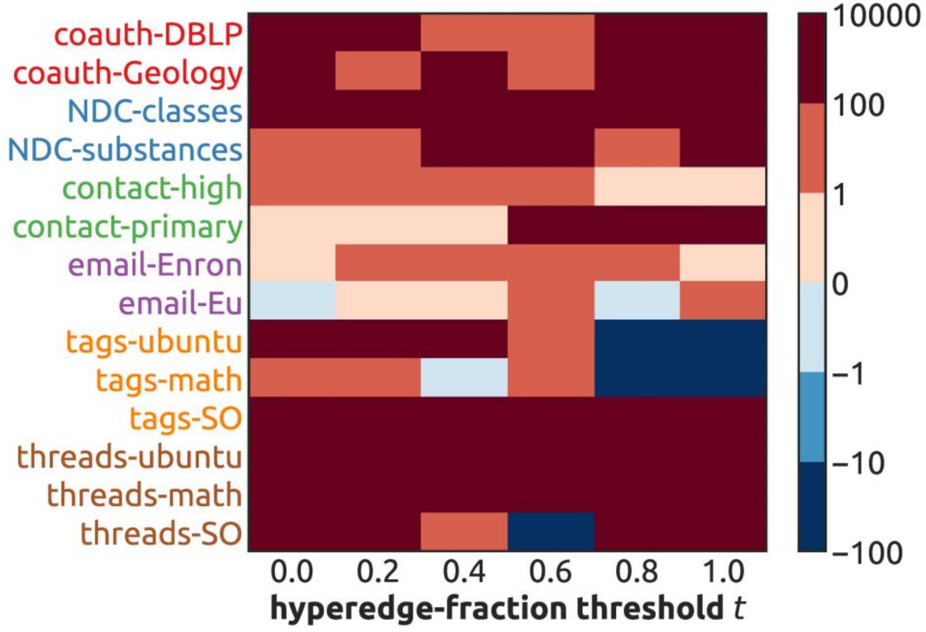
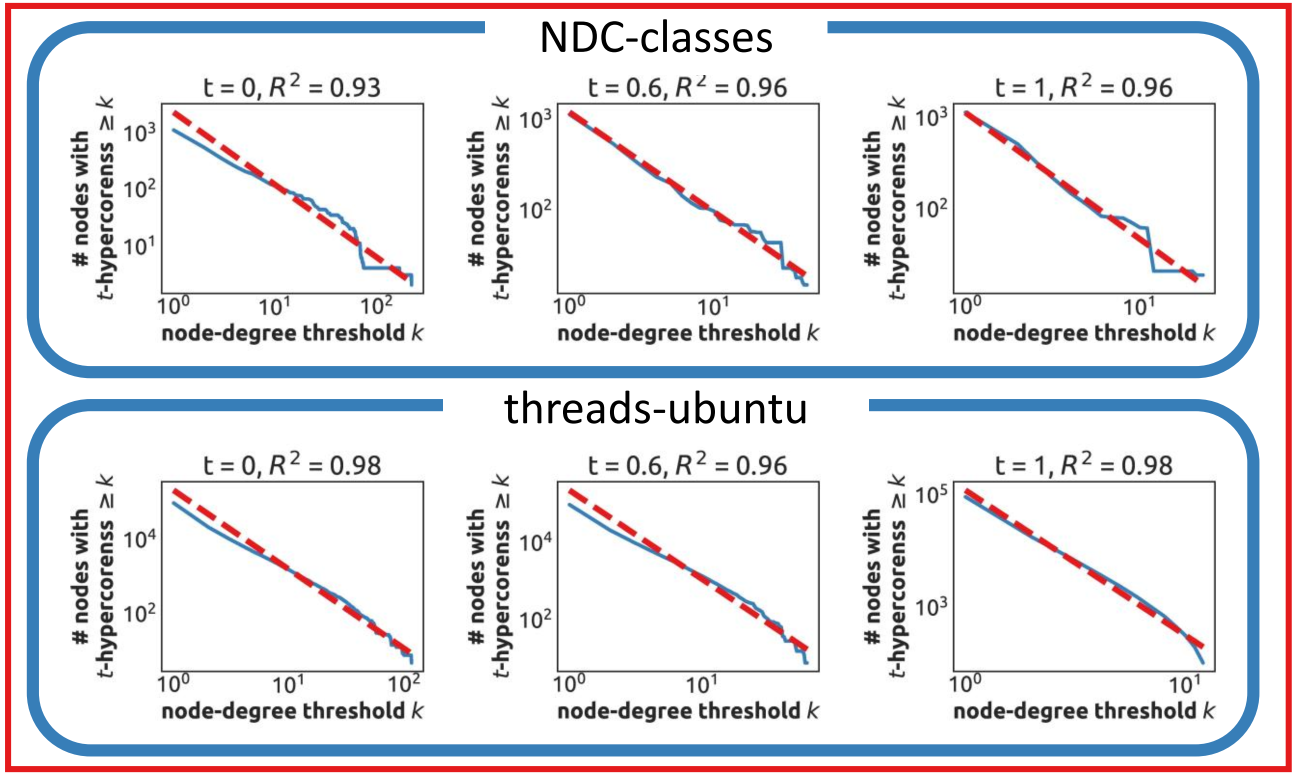
- **Right: Red dashed lines** are reference power-law fitting lines



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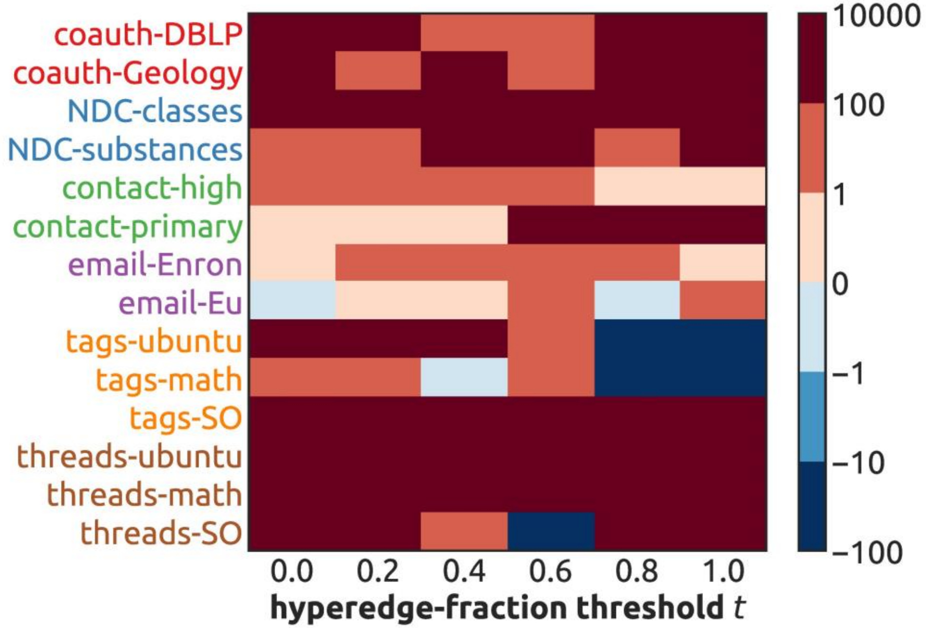


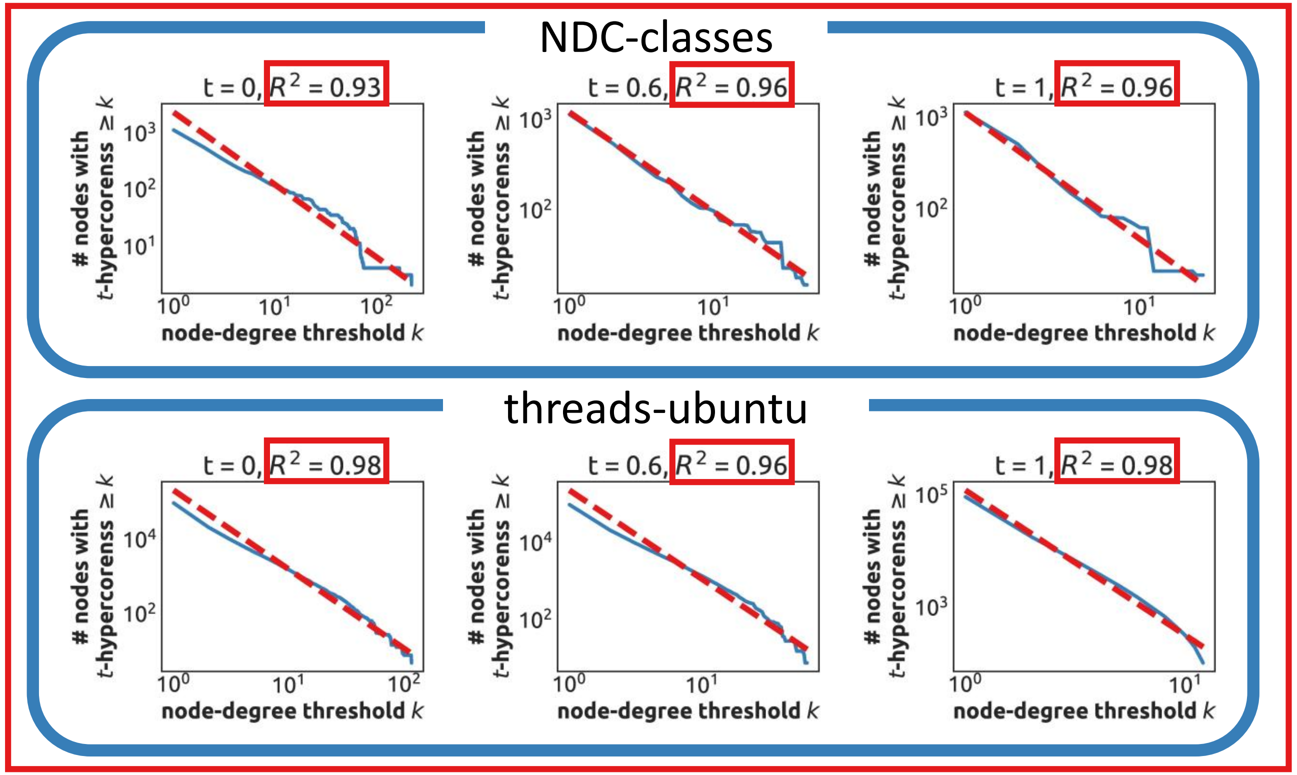

# Observation 3: $t$-Hypercoreness is Different

- **Observation:** $t$-Hypercoreness is statistically different **from other existing centrality measures**
  - **Meaning:** t-Hypercoreness can provide unique insights of a hypergraph
- **Observation:** Even for the same hypergraph, with **varying $t$ values**, the $t$-hypercoreness can be statistically different **from each other**
  - **Meaning:** Using different t values can provide us different insights

# Roadmap

- Concepts & Algorithms
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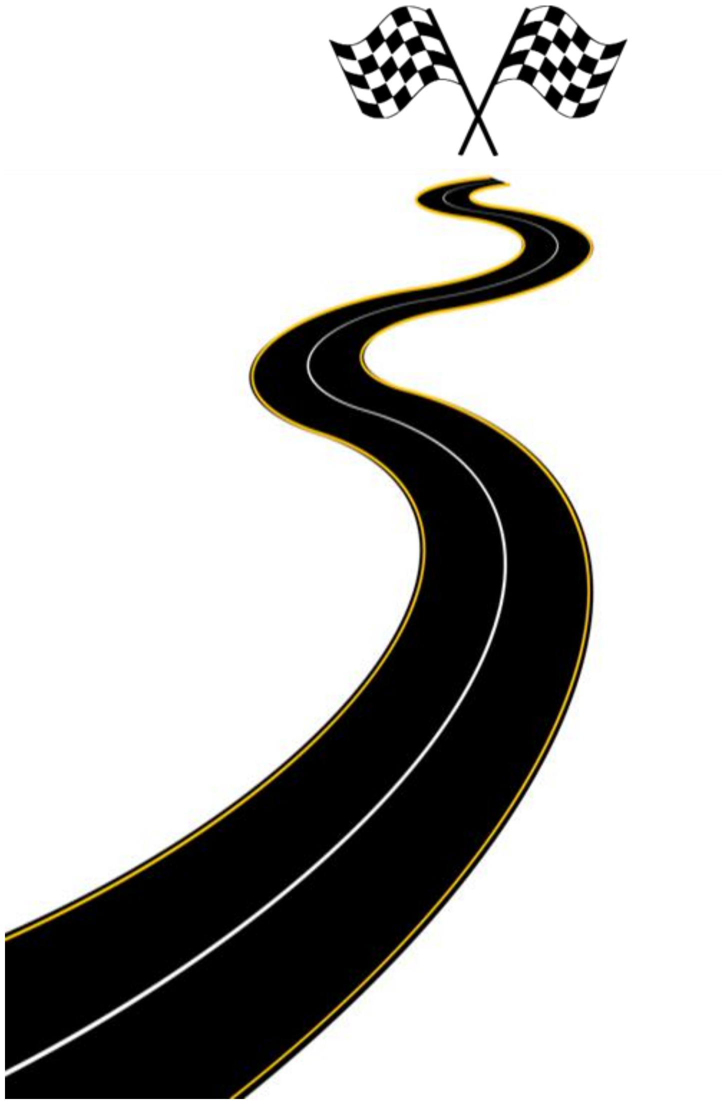
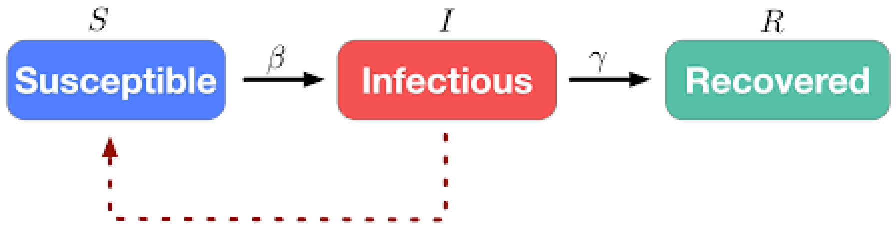
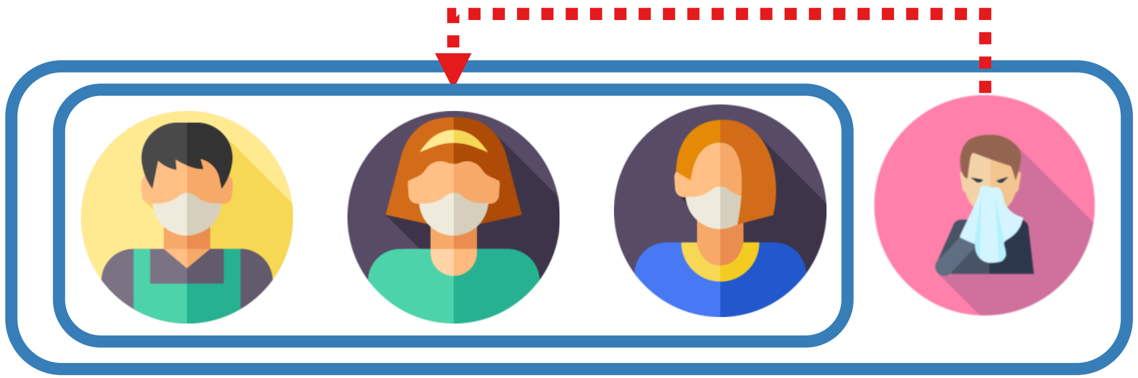
- **Applications <<**
- Conclusions



# Application 1: Influential-Node Identification

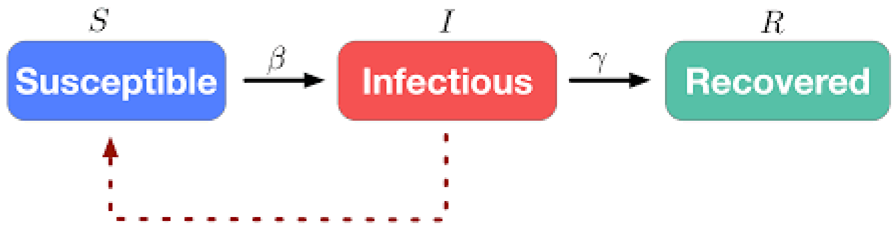
- **Spoiler:** In real-world hypergraphs, $t$-hypercoreness with a proper $t$ value **identifies influential nodes** well
- **Set-up:** We use a widely-used **epidemic model**, the SIR model
  - A **single initially infected node**
  - Infected nodes can infect susceptible nodes **in the same hyperedge**
  - Infected nodes have some probability to **recover** (and become immune)
  - The process terminates with only **susceptible** and **recovered** nodes

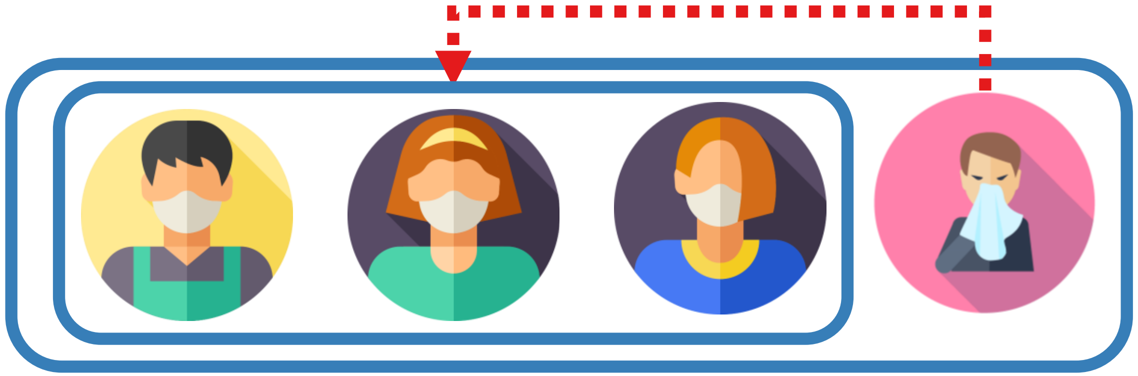


# Application 1: Influential-Node Identification

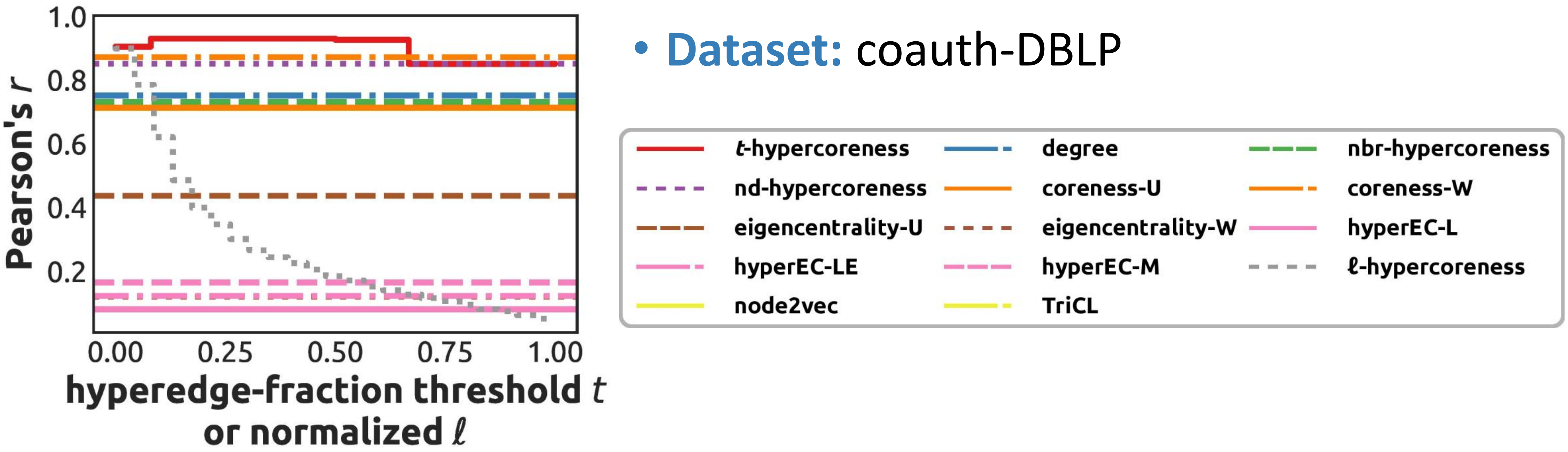
- **Spoiler:** In real-world hypergraphs, $t$-hypercoreness with a proper $t$ value **identifies influential nodes** well
- **Influence:** We measure the **influence** of each node $v$ as the number of **ever-infected nodes** when $v$ is the initially infected node
  - **Ever-infected nodes:** the nodes in the recovered state in the end
  - The influence of each node is measured by **simulations**



# Application 1: Results

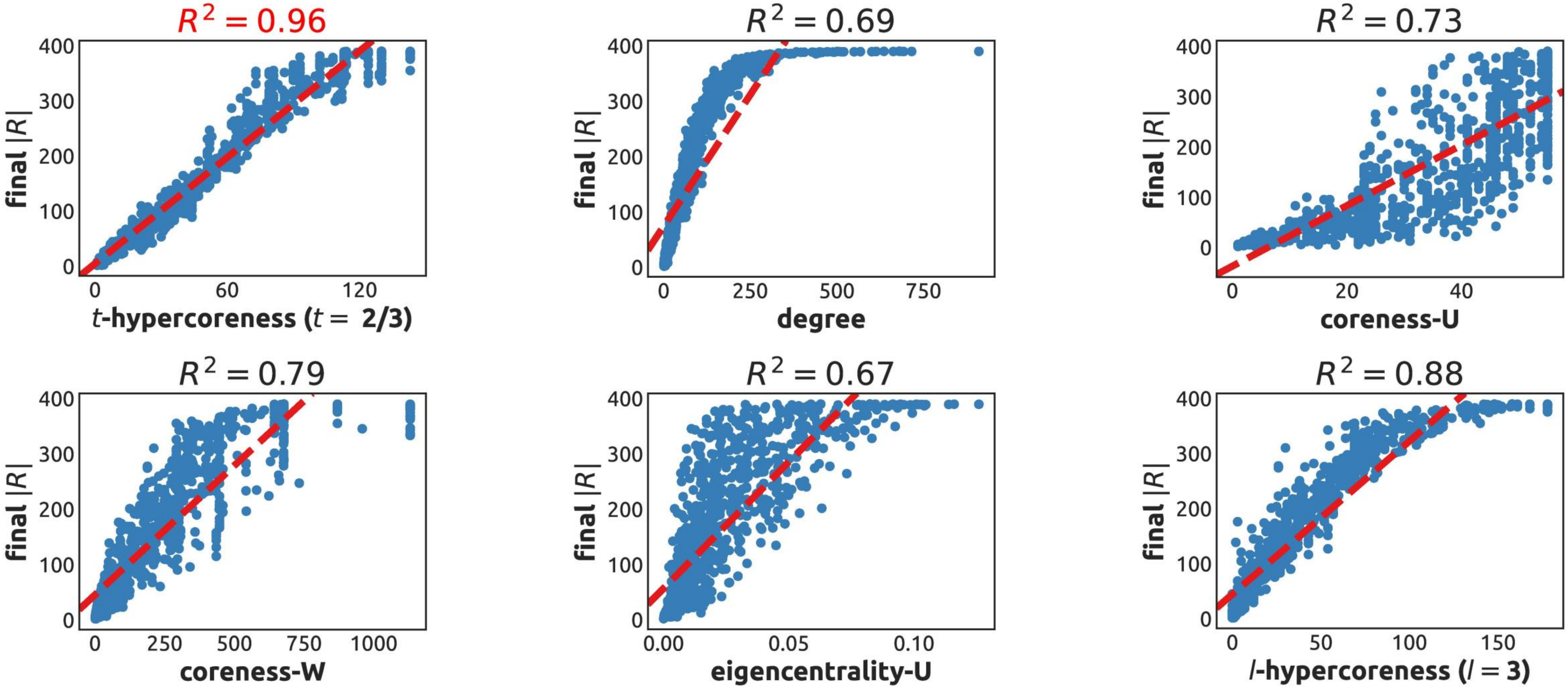
- **Metric:** For each considered measure, we compute the **Pearson's r between** the values of the **measure** and the **influences** of the nodes
  - The higher Pearson's r, the better
- **Result summary:** $t$-Hypercoreness with a proper $t$ usually has **high correlation** with the influences of nodes




- **Dataset:** coauth-DBLP

# Application 1: Results

- **Dataset:** email-Eu

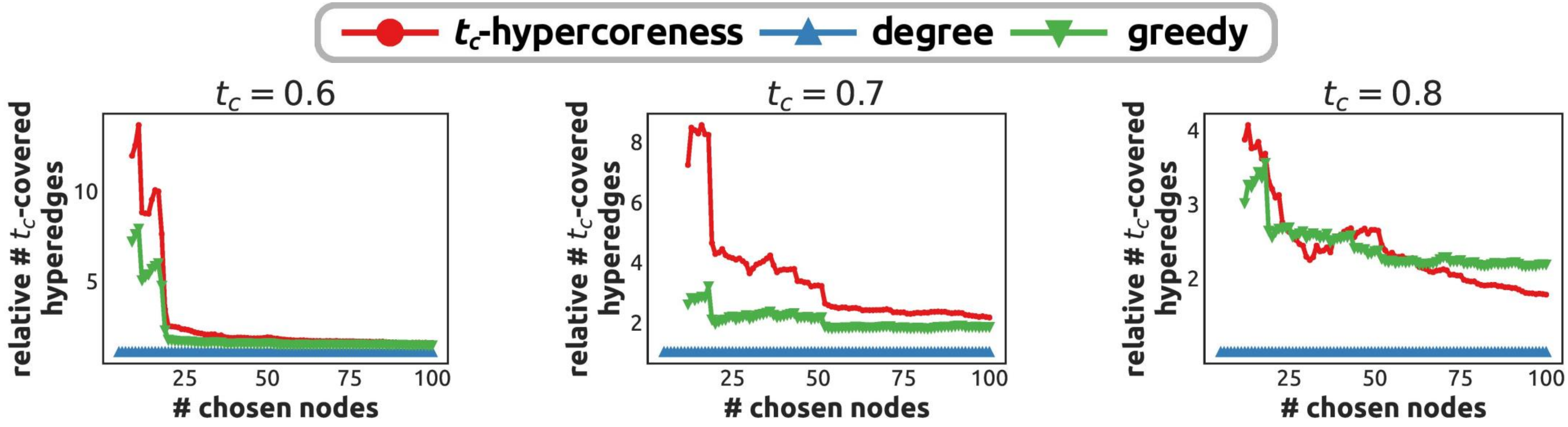


# Application 2: Dense Substructure Discovery

- **Spoiler:** $(k, t)$-Hypercores can be used to efficiently and effectively **find dense substructures**
- **Set-up:** We consider a **vertex cover** problem
  - **Given:** A hypergraph $H$, # nodes to choose $k_c$, and the cover threshold $t_c$
  - **Aim to:** Choose $k_c$ nodes to maximize # covered hyperedges
  - A hyperedge is **covered** if $\geq t_c$ proportion of its constituent nodes are chosen
- **Considered methods:**
  - **$t_c$-Hypercoreness:** The $k_c$ nodes with the highest $t_c$-hypercoreness
  - **Degree:** The $k_c$ nodes with the highest degree
  - **Greedy:** Greedily increases the number of covered hyperedges

# Application 2: Results

- We vary $k_c$ from 10 to 100 and vary $t_c \in \{0.6, 0.7, 0.8\}$
- The performance of the **degree** method is used as the reference one
- **Result summary:** Overall, $t_c$-hypercoreness outperforms the other two methods, i.e., covers more hyperedges
- Below are the results averaged over all the datasets



# Application 3: Hypergraph Vulnerability Detection

- **TL;DR:** The concept of $(k, t)$-hypercores can be used to detect **vulnerability** in hypergraphs
- We consider an **optimization problem** on hypergraphs using the concept of $(k, t)$-hypercores
- We aim to remove a small number of nodes from a given hypergraph so that the size of a $(k, t)$-hypercore is minimized
  - Such nodes are **vulnerable nodes** in the hypergraph
  - We can pay attention to and **protect** such nodes in real-world applications

# Roadmap

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# Conclusions

- Our contributions are summarized as follows:

✓**Novel concepts** generalizing $k$-cores to hypergraphs, with

✓**Theoretical properties** and **practical computational algorithms**

✓**Various observations** utilizing the proposed concepts

✓**Extensive applications** showing the usefulness of the proposed concepts

**Code:** bit.ly/hypercore_code